KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 504 : TOPOLOGY

DATE: THURSDAY 5TH APRIL 2012 TIME: 9.00 A.M. – 12.00 P.M.

INSTRUCTIONS
Attempt Question ONE and any other TWO Questions.

1. (a) (i) Define the Hansdorff property in a topological space. (1 Mark)

(ii) Let X be Hansdorff. If $f: X \to Y$ is a closed bijection, show that Y is also Hansdorff. (4 Marks)

(b) (i) State what is meant by a regular space. (1 Mark)

(ii) Let Y be regular. Show that for each $y \in Y$ and nbd of U of y, there exists a nbd V of y with $y \in V \subset \overline{V} \subset U$. (4 Marks)

(c) (i) State what is meant by a normal space. (1 Mark)

(ii) Let X be normal and let $f: X \to Y$ be a continuous closed surjection. Show that Y is normal. (4 Marks)

(d) Show that a discrete space is compact if and only if it is infinite. (3 + 2 Marks)

(e) Show that the continuous image of a compact set is compact. (5 Marks)

(f) (i) Define countable compactness. (2 Marks)

(ii) Give an example of countable compact space. (3 Marks)

2. (a) Let Y be Hansdorff. If $p \in Y$, show that for each $q \in Y$, $p \neq q$ there exists open $U(p)$ such that $q \in \overline{U}(p)$. (6 Marks)

(b) Use 2(a) or otherwise to show that for each $p \in Y$, $\cap\{\overline{U} \mid U$ is a nbd of $p\} = p$. (7 Marks)

(c) Show that each finite set in a Hansdorff space is closed. (7 Marks)

3. (a) Let Y be normal. Show that for each closed $A \subset Y$ and open U, $A \subset U$ that is an open V such that $A \subset V \subset \overline{V} \subset U$. (8 Marks)

(b) For such pair of disjoint closed sets A, B, show that there is an open U with $A \subset U$ and $\overline{U} \cap B = \phi$. (7 Marks)

(c) Show that each pair of disjoint closed sets have nbds whose closures do not intersect. (5 Marks)

4. (a) Show that a space Y is compact iff the finite intersection property holds in Y. (4 + 4 Marks)

(b) Show that a compact subset A of a Hansdorff space X is closed in X. (6 Marks)

(c) Show that a finite union of compact subsets of a Hansdorff space is compact. (6 Marks)

5. Let X be arbitrary, Y Hansdorff and $f, g : X \to Y$ continuous.

(a) Show that $\{x : f(x) = g(x)\}$ is closed. (8 Marks)

(b) If $D \subset X$ is dense and $f \mid D = g \mid D$ then $f = g$ on X. (7 Marks)

(c) (i) Define the term identification topology. (2 Marks)

(ii) If $\gamma(p)$ is the identification topology determined by p, show that $\gamma(p)$ is a topology. (3 Marks)