



KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

SMA 506: GROUP THEORY I

DATE: Thursday 29th March, 2012

TIME: 9.00 am – 12.00 pm

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions.

QUESTION 1

- a) Give the subgroup structure of the quaternion group. (3 marks)
- b) Give two composition series of $\mathbb{Z}_{20}$. (2 marks)
- c) Give all the factors groups of the subnormal series $\{0\} \triangleleft 72\mathbb{Z} \triangleleft 24\mathbb{Z} \triangleleft 8\mathbb{Z} \triangleleft 4\mathbb{Z} \triangleleft \mathbb{Z}$ showing which known groups these factor groups are isomorphic to. (5 marks)
- d) i) Define the centre of a group (2 marks)
ii) Find the centre of the quaternion group. (2 marks)
- e) i) Show whether or not S_5 is soluble. (3 marks)
ii) Is S_5 nilpotent? Give reasons. (3 marks)
- f) Let G_m denote the set of 2×2 matrices of the form $\begin{pmatrix} \alpha & \beta \\ 0 & 1 \end{pmatrix}$, where α and β are elements of $\mathbb{Z}_3$ and $\alpha \neq 0$. Given that G_m is a group with respect to the usual operation of matrix multiplication.

- i) Find the order of G_m . (2 marks)
- ii) Write down explicitly the matrices belonging to the group G_m (3 marks)
- iii) Find the inverse of each of matrices in (ii). (3 marks)
- g) Find the order of $(3, 2)$ in $\mathbb{Z}_{12} \times \mathbb{Z}_4$ (2 marks)

QUESTION 2

- a) Let α and β denote the functions defined on a suitable subset of real numbers by

$$\alpha(x) = \frac{1}{x}, \beta(x) = \frac{1}{1-x}.$$
 - i) Find the orders of α and β . (5 marks)
 - ii) Find $\langle \alpha, \beta \rangle$, the group generated by α and β . Which well known group is this group isomorphic to? (5 marks)
- b)
 - i) Find all the elements in A_4 . (3 marks)
 - ii) Find all the elements in D_6 . (3 marks)
- c)
 - i) Give two arguments showing that $\mathbb{Z}_4$ is not isomorphic to the Klein 4 – group V . (2 marks)
 - ii) Give two arguments showing that the dihedral group D_4 is not isomorphic to the quaternion group Q . (2 marks)

QUESTION 3

- a) Explain the difference between a subnormal series and a normal series of a group G . (2 marks)
- b)
 - i) Explain what is meant by a refinement of a subnormal series of a group G . (2 marks)
 - ii) Find a refinement of the subnormal series $\{0\} \triangleleft \langle 3 \rangle \triangleleft \mathbb{Z}_{12}$ (2 marks)
- c)
 - i) Define a soluble group. (2 marks)
 - ii) Show that p -groups are soluble. (4 marks)

- d) State with reasons whether the following groups are soluble or not.
- i) $A_n, n \geq 5$ (2 marks)
 - ii) $S_n, n \geq 5$ (2 marks)
 - iii) $D_n, n \geq 5$ (2 marks)
- c) Differentiate between a direct product and a semidirect product group. (2 marks)

QUESTION 4

- a)
 - i) Define a nilpotent group. (2 marks)
 - ii) Show that every abelian group is nilpotent. (3 marks)
- b) Show that a nilpotent group is soluble. (3 marks)
- c) Find a lower central series for the following groups.
 - i) $A_n, n \geq 5$ (2 marks)
 - ii) $S_n, n \geq 5$ (2 marks)
 - iii) $D_n, n \geq 5$ (2 marks)
- d) Find the upper central series for the groups in part (c). (4 marks)
- c) List all the abelian groups of order 540. (2 marks)