KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 508: COMPLEX ANALYSIS I

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DATE: MONDAY 2ND APRIL 2012 **TIME: 9.00 A.M. – 12.00 NOON**

INSTRUCTIONS

Answer ANY three Questions

Question One (23 marks)

a) i) Explain the difference between conformal and isogonal transformations. (3 marks)

ii) Discus the conformality of $f(z) = z^2$ at the point (4 , 1) , the point of intersection of the lines y = 1 and y = x - 3 . (10 marks)

b) Show that under the transformation $w = \frac{1}{z}$, the image of the lines y = x + 2 and y = 0 are the circle $u^2 + \frac{1}{2}u + v^2 + \frac{1}{2}v = 0$ and the line v = 0 respectively.

Sketch the four curves, determine corresponding directions along them, and verify conformality of the mapping at the point z = (- 2 , 0) (10 marks)

Question Two (23 marks)

a) State and prove the Maximum Mudulus Theorem. (7 marks)

b) Show that the angle of rotation at the point $Z_o = 1 + i\sqrt{3}$ under the transformation $f(z) = Z^5$ is $\frac{4}{3}\pi$. Determine the linear and area scale factors. (5 marks)

c) State Rouche's Theorem. (3 marks)

d) Use Rouche's theorem to prove that

i) Every polynomial of degree n has n zeros, counting multiplicities. (4 marks)

ii) All the roofs of $z^8 + 15z^2 - 10z + 82 = 0$ lie between $|z| = 2$. (4 marks)

Question Three (23 marks)

a) Explain the concept of analytic continuation. (3 marks)

b) Prove that $f_2(z) = \dfrac{1}{1+i} \displaystyle\sum_{n=0}^{\infty} \left(\frac{z+i}{1+i}\right)^n$ is an analytic continuation of $f_1(z) = \displaystyle\sum_{n=0}^{8} z^n$, showing graphically the regions of convergence of the series. (7 marks)

c) State and prove Schwartz's Reflection principle. (13 marks)

Question Four (23 marks)

a) i) Define a linear fractional transformation. (2 marks)

ii) Find a linear fractional transformation which maps the points *2i* , - *2i,* I of z-plane onto *0, 2,* ∞ of the w-plane respectively. (6 marks)

iii) Show that every linear fractional transformation with exception of the identity transformation I (z) = z, has at most two fixed points in the extended z-plane. (6 marks)

b) State without proof the Argument Theorem. (3 marks)

c) Use the argument Theorem to evaluate $\int_c \frac{f^1(2)}{f(2)}\, dz$ when

i) $f(2) = z^5 - 3i\, z^2 + 2z - 1 + i$ and c encloses all zeros of $f(z)$.

(2 marks)

ii) $f(z) + \dfrac{(z^2+1)}{(z^2+2z+2)^3}$ and c is the circle $|z| = 4$.

(4 marks)

Question Five (23 marks)

a) Define a gamma function and prove the recursive property of the gamma function

i.e. $\Gamma(z+1) = z\Gamma z$ for $R_e\, z > 0$ (6 marks)

b) Use the gamma function to evaluate each of the following integrals

i) $\int_o^\infty \left(y\, e^{-y2}\right)^{1/4} dy$ (5 marks)

ii) $\int_0^\infty u^{3/2}\, e^{-3u}\, du$ (5 marks)

c) Prove tht $B(m,n) = \dfrac{\Gamma(m)\ \Gamma(n)}{\Gamma(m+n)}$, where $B(m, n)$

and $\Gamma(m)$ are beta and gamma functions respectively. (4 marks)

d) Prove that $\sin h\,(z+\pi i) = -\sin h\ z$. (3 marks)