KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL/ELECTRONICS ENGINEERING

AND CIVIL ENGINEERING

ECU 106: ENGINEERING ALGEBRA II

DATE: THURSDAY, 12TH APRIL 2011 **TIME: 11.00 A.M – 1.00 P.M**

INSTRUCTIONS: Answer QUESTION 1 and ANY OTHER two questions.

QUESTION 1

(a) Given that $A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$, determine the matrix C such that

$C = (A + B)^2 - (A^2 + 2AB + B^2)$.

(5 Marks)

(b) Let

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 0 \\ 1 & 4 & 2 \end{pmatrix}.$$

Find A^{-1}.

(5 Marks)

(c) Solve the system

$$2x + y - 2z = 10$$

$$3x + 2y + 2z = 1$$

(5 Marks)

(d) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\begin{pmatrix} -3 & 1 & 2 \\ -2 & 0 & 2 \\ -2 & 1 & 1 \end{pmatrix}$$

(5 Marks)

(e) Find an equation for the plane that contains the point $(2, 1, -1)$ and perpendicular to the plane $2x - 3y + z = 1$. (5 Marks)

(f) Use Boolean algebra to simplify

$$(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge q)$$

(5 Marks)

QUESTION 2

(a) Show that the lines

$$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{4} \quad \text{and} \quad \frac{x+2}{4} = \frac{y}{-3} = \frac{z-\frac{1}{2}}{-1}$$

intersect and find the point of intersection. (7 Marks)

(b) (i) Find the symmetric equations of the line through the points $(-1, 3, 7)$ and $(4, 2, -1)$ (3 Marks)

(ii) Find the distance between the lines

$$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{4} \quad \text{and} \quad \frac{x+2}{4} = \frac{y}{-3} = \frac{z+1}{1} .$$ (6 Marks)

(c) Find the angle between the planes $3x + y - z = 5$ and $x - 6y - 2z = 10$.

(4 Marks)

QUESTION 3

(a) (i) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $b \neq 0$, $c \neq 0$ and $a \neq d$. Given that $\begin{pmatrix} e & bm \\ cm & f \end{pmatrix}$ commutes with A , show that $m = \dfrac{e-f}{a-d}$. (4 Marks)

(ii) If $A = \begin{pmatrix} 1 & 6 \\ -3 & 4 \end{pmatrix}$, determine a matrix in terms of e and f only which commutes with A.

(3 Marks)

(b) Use the Cramer's rule to solve

$$2x - 5y + 2z = 7$$

$$x + 2y - 4z = 3$$

$$3x - 4y - 6z = 5$$

(5 Marks

(c) (i) Determine whether or not $(2, -5, 3)$ is a linear combination of $(1, -3, 2)$, $(2, -4, -1)$ and $(2, -5, 7)$.

(4 Marks

(ii) For what value of k will the vector $(1, -2, k)$ be a linear combination of the vectors $(3, 0, -2)$ and $(2, -1, -5)$?

(4 Marks)

QUESTION 4

(a) Reduce the matrix

$$\begin{pmatrix} 6 & 3 & -4 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{pmatrix}$$

to echelon form.

(6 Marks)

(b) For what values of x is

$$A = \begin{pmatrix} 1 & x & x-1 \\ 0 & 1 & -1 \\ x & 1-x & 2x-1 \end{pmatrix}$$

not invertible?

(7 Marks)

(c) Find x, y, z and w if

$$3\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+w & 3 \end{pmatrix}.$$ (7 Marks)

QUESTION 5

(a) Given that $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$.

Find (i) Eigenvalues of A (3 Marks)

(ii) Eigenvectors of A (3 Marks)

(b) Use Boolean algebra to simplify the following

(i) $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ (4 Marks)

(ii) $\sim[(p \wedge q) \vee \sim p]$ (3 Marks)

(c) Verify whether each of the following is a tautology, a contradiction or neither.

(i) $(p \vee q) \Rightarrow \sim p$ (3 Marks)

(ii) $p \vee \sim (p \wedge q)$ (2 Marks)

(d) Find the inverse of the converse of $\sim p \Rightarrow q$ (2 Marks)