KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

INSTITUTIONAL BASED PROGRAMME (AUGUST SESSION)

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 509: COMPLEX ANALYSIS II

DATE: **Tuesday 3rd January 2012** **TIME:** **9.00a.m -12.00p.m**

Instructions: *Answer **Question 1** and any other **TWO** questions.*

QUESTION ONE (30 marks)

a) If $f(z)$ is an entire function with zeros α_1 and $\alpha_2, \alpha_1 \neq \alpha_2$ of multiplicities m_1 and m_2, prove that $f(z) = e^{g(z)}(z-\alpha_1)^{m_1}(z-\alpha_2)^{m_2}$ where $g(z)$ is an entire function. (6 marks)

b) Prove that $\prod_{p-prime}\left(1-\frac{1}{p^2}\right) = \frac{\pi^2}{6}$ (6 marks)

c) State Jensen's inequality and use Jensen's formula to prove it. (6 marks)

d) If $\tan z = 2z\sum_{k=0}^{\infty}\frac{1}{\frac{(2k+1)^2}{4}\pi^2 - z^2}$, show that $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots = \frac{\pi^2}{8}$ (6 marks)

e) Show that a necessary and sufficient condition that $\prod_{k=1}^{\infty}\left(1+|w_k|\right)$ converges is that $\sum_{k=1}^{\infty}|w_k|$ converges. (6 marks)

QUESTION TWO (20 marks)

a) State and prove the Mittag Loffler's expansion theorem. (10 marks)

b) Use the Mittag Loffler's expansion theorem to show that
$\cot z = \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{1}{z^2 - n^2\pi^2}$. (10 marks)

QUESTION THREE (20 marks)

a) Prove that $\sin(\pi z) = \pi z\prod_{k=1}^{\infty}\left(1-\frac{z^2}{k^2}\right)$ and use it to conclude that

$\sin z = z\prod_{k=1}^{\infty}\left(1-\frac{z^2}{k^2\pi^2}\right)$ (8 marks)

b) Use a) above to prove that $\cos z = \prod_{k=1}^{\infty}\left(1-\frac{4z^2}{(2k-1)^2\pi^2}\right)$ (6 marks)

c) Let $f(z)$ be a non-constant bounded analytic function on D(0, 1) (the unit disk) and $a_1, a_2, a_3, \cdots$ are zeros of $f(z)$ counted according to multiplicities, prove that $\sum_{j=1}^{\infty}\left(1-|a_j|\right)<\infty$ (6 marks)

QUESTION FOUR (20 marks)

a) State and prove **Jensen's formula.** (10 marks)

b) Suppose $f(z)$ is a non-constant entire function of finite order, then the image of f contains all complex numbers except possibly one. (10 marks)

QUESTION FIVE (20 marks)

a) Define the Riemann **Zeta function** and show that $\xi(z) = \prod_{p-prime}\left(1-\frac{1}{p^z}\right)^{-1}$ (10 marks)

b) Define the **Gamma function** and prove that $\Gamma(z)\xi(z) = \int \frac{t^{z-1}}{e^t-1}dz$ (10 marks)

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