



KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2009/2010
INSTITUTIONAL BASED PROGRAMME (IBP)
EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 531: ORDINARY DIFF. EQUATIONS II

DATE: Wednesday, 28th April, 2010

TIME: 9.00 a.m. – 12.00 p.m.

INSTRUCTIONS:

Answer question **ONE** and any **TWO** of the remaining questions. Show all the necessary working.

QUESTION ONE: (30 MARKS)

- a) Show that $y = 4e^{2x} + 2e^{-3x}$ is a solution of the initial - value problem

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 6y = 0,$$

$$y(0) = 6$$

$$y'(0) = 2.$$

Is $y = 2e^{2x} + 4e^{-3x}$ also a solution of this problem? Explain why or why not.

(5 marks)

- b) Solve the initial - value problem

$$y'' - 2y' - 3y = 2e^x - 10\sin x$$

$$y(0) = 2 ; y'(0) = 4$$

by using the method of undetermined coefficients.

(10 marks)

- c) i) Define a boundary – value problem.
 ii) Solve the boundary – value problem

$$y'' + \pi^2 y = 0$$

$$y(0) = 0 ; y(1) = 0$$

(9 marks)

- d) i) Define a critical point of an autonomous system.
 ii) The equation

$$\frac{d^2 x}{dt^2} = x^5 - 5x^4 + 5x^3 + 5x^2 - 6x$$

is the differential equation of a non – linear conservative system. Find the equivalent non – linear autonomous system and its critical points.

(6 marks)

QUESTION TWO: (20 MARKS)

- a) Determine whether the boundary – value problem

$$\frac{d^2 y}{dx^2} + \lambda y = 0,$$

$$y(0) = 0; y(\pi) = 0$$

is a Sturm – Liouville problem or not.

(3 marks)

- b) Find the characteristic values and functions of the Sturm – Liouville problem

$$\frac{d}{dx} \left[x \frac{dy}{dx} \right] + \frac{\lambda}{x} y = 0$$

$$y'(1) = 0 ; y'(\ell^{2\pi}) = 0,$$

where the parameter λ is non – negative.

(11 marks)

- c) Transform

$$y'' + 2xy' + (x + \lambda)y = 0$$

into the form required for Sturm – Liouville problem.

(6 marks)