KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

ECU 203: ENGINEERING MATHEMATICS VIII

DATE: FRIDAY 13TH APRIL 2012 **TIME: 8.00 A.M. – 10.00 A.M.**

Instructions: Answer any **three** questions

Question One

(a) Find the Laplace transform of

(i) $t^3 + e^{4t}$ (1 mark)

(ii) sin 3t + cos 4t (1 mark)

(b) Find the Laplace inverse transform of

(i) $\frac{s-4}{s^2-4}$ (3 marks)

(ii) $\frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{s^3}$ (2 marks)

(iii) $\frac{6s-4}{s^2-s}$ (3 marks).

(c) Solve $y'' - 4y' + 4y = 0, y(0) = 0, y'(0) = 2$ (10 marks).

Question Two

(a) The temperature $u(x,t)$ in a semi- infinite rod $0 < u < \infty$ is determined by the differential equation.

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

And the conditions

(i) $u = 0$ when $t = 0,\ u \geq 0$

(ii) $u = u_0 = \text{constant}$, when $x = 0$ and $t > 0$

Making use of sine transform, show that

$$u(x,t) = \frac{2}{\pi} u_0 \int_0^{\infty} \frac{\sin \omega x}{\omega} \left(1 - e^{-Kw^2 t}\right) dw$$

(20 marks)

Question Three

a) (i) Use Fourier series to show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}$ (8 marks)

(ii) Find the half cosine series of the function

$$\text{f (x)} = \begin{cases} x & 0 < x < \pi \\ 0 & -\pi < x < 0 \end{cases}$$

(5 marks)

(b) (i) Find the Fourier integral representation of

$$\text{f (x)} = \begin{cases} 1 & 0 < x < 3 \\ 0 & x > 3 \end{cases}$$

(3 marks)

(ii) Find the Fourier transform of the function

$$\text{f (x)} = \begin{cases} k & 0 < x < a \\ 0 & a > 0 \end{cases}$$

(3 marks)

(iii) Find $\Gamma(\frac{5}{2})$ (3 marks)

Question Four

a) Evaluate (i) $t * e^t$ (3 marks)

(ii) $1 * \sin t$ (3 marks)

b) Use the convolution theorem to evaluate

i. $\frac{1}{s(s-2)}$ (3 marks)

ii. $\frac{1}{(s^2+1)^2}$ (3 marks)

c) Using the Laplace transformation, solve

i. $y(t) = 1 + \int_0^t y(\tau) d\tau$ (4 marks)

ii. $y(t) = sin2t + \int_0^t y(\tau) sin2(t - \tau) d\tau$ (4 marks)

Question Five

Find the temperature $u(x,t)$ in a laterally insulated copper bar length, $L = \pi\ m$ if the initial temperature f(x)=$10^0\,C$ and the ends are kept at $0^0\,C$ and $c^2 = 1$. (20 marks)