KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2009/2010

INSTITUTIONAL BASED PROGRAMME

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 535: NUMERICAL ANALYSIS II

DATE: THURSDAY 30TH DECEMBER 2010 TIME: 9.00 A.M. - 12.00 NOON

INSTRUCTIONS

Answer Any THREE Questions

Question 1

a) Derive the general quadratine formula

$$\int_{xo}^{xy} f(x)\,dx = nh\left[y_o + \frac{n}{2}\Delta yo\,\frac{n(2n-3)}{12}\Delta^2 yo + \frac{n(n-2)^2}{24}\Delta^3 yo + --\right]$$

b) i) Using the quadrative formulae in (a) above derive the Boole's Rule.

(5 marks)

ii) Evaluate $\int_o^4 \frac{dx}{1+x^2}$ using Boole's rule taking h = 1 and h = 0.5 . Compare the results with the actual value and indicate the errors in both. (10 marks)

c) Calculate $\int_0^{1/2} \frac{x}{\sin x}\,dx$ using 4 term of the Taylors expression of the integrand.

(3 marks)

Question 2

a) i) Using Euler-Maclauriau's formula, obtain the value of

$$\log_e^2 \quad \text{from} \int_0^1 \frac{1}{1+x}\,dx$$ (5 marks)

ii) Using Euler-Maclaurius formula to prove that

$$\sum_i^n x^2 = \frac{n(n+1)\,(2n+1)}{6}$$ (5 marks)

b) Find the values of a, b and c such that the truncation error in the formula

$$\int_{-h}^{h} f(x)dx = h\,\{a\,f(-h) + b\,f\,(o) + a\,f\,(h)\}$$
$$+ h^2\,c\left[f'\,(-h) - f'\,(h)\right]$$

is minimized.

Suppose that the composite formula has been used with the step length h and $h/2$, giving I (h) and I ($h/2$). State the result of using Richardson's extrapolation in this value. (13 marks)

Question 3

a) Compute an approximation to y(1), $y'(1)$ and $y'''(1)$ with Taylor's algorithm of order two and step length h = 1 when y(x) is the solution to the initial- value problem.

$$y''' + 2y'' + y' - y = \cos x\,,\ 0 \le x \le 1\,,\ y(0) = 0 \quad y'(0) = 1\,,\ y''\,(0) = 2$$

(8 marks)

b) Consider the following Range-Kutta method for the d.e.

$$y' = f(x, y)$$

$$W_{1+1} = w_i + \tfrac{1}{6}(k_1 + 4k_2 + k_3)$$

$$K_1 = h\, f(x_i, w_i)$$

$$K_2 = h\, f(k_i + \tfrac{h}{2},\ w_i + \tfrac{k}{2})$$

$$K_3 = h\, f(k_1 + h,\ w_i - k_1 + 2k_2$$

What is the result after one step of length h when $y' = y$, $y(0) = 1$

(8 marks)

c) i) Give the exact solution of the wP $y' = xy \quad y(0) = 1$

ii) Estimate the error at x = 1, when Euler's method is used, with step size h = 0.1. (4 marks)

Question 4

a) Solve the system of equation

$$\frac{dy}{dx} = yz + x \quad , \quad \frac{dz}{dx} = xz + y \ ;$$

given that y (0) = 1 , z (0) = -1 for y (0.1), z (0.1). (12 marks)

b) Solve the Laplace equation

$$\frac{d^2 u}{dx^2} + \frac{d^2 u}{dy^2} = 0$$

By employing the five-point formulae, which satisfy the following Dirichlet bounday condition

$$u(0, y) = 0 \quad , \quad u(x, 0) = 0$$

$$u(x, 1) = 100x \, , \quad u(1, y) = 100y$$ (11 marks)

Question 5

a) Solve the wave-equation

$$\frac{d^2y}{dt^2} = \frac{d^2u}{dx^2} \quad , \quad 0 \le x \le 1$$, subject to the initial- condition

$u(x, 0) = x^2$, $\frac{du}{dt}(x, 0) = 1$ and the boundary condition $u(0, t) = 0$,

$u(1, t) = 1 + t$, $t > 0$ by taking $h = k = 0.2$ and compute the solution for the first four time steps, use the central difference approximation for the initial condition.

(13 marks)

b) Evaluate the double integral $I = \int_1^2 \int_1^2 \frac{dx\,dy}{x + y}$ by using trapezoidal rule with

$h = k = 0.25$. (10 marks)