KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

INSTITUTION BASED PROGRAMME

AUGUST SESSION EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 535: NUMERICAL ANALYSIS II

DATE: Thursday 29th December, 2011 **TIME:** 9.00 a.m. – 12.00 p.m.

INSTRUCTIONS*: Answer any THREE questions.*

Question 1

(a) Find the quadrature formulae

$$\int_0^{\prime} f(x)\ \frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \ \propto_2\ f\left(\tfrac{1}{2}\right) + \propto_3 f(1)$$

Which is exact for polynomial of highest possible degree then use the formula on

$\int_0^1 \frac{1}{\sqrt{x - x^3}}\,dx$ and compare with the exact value. [13 marks]

(b) Using modified Euler's method, obtain the solution of the differential equation

$y' = t + \sqrt{y},\ y(0) = 1,$ for the range $0 \le t \le 0.6$ in steps of 0.2. [6 marks]

(c) Compute by using Taylors expansion

$\int_{0.0}^{0.2} \frac{x^2}{\text{Cosx}}\,dx$ [4 marks]

Question 2

(a) (i) Using the method of undetermined coefficient find the nodes and weight of the quadrature formulae

$$\int_0^{\infty} e^{-x}\ f(x)\,dx = \lambda_0\ f(x_0) + \lambda_1\ f(x_1)$$

Use the result $\int_0^{\infty} x^n\, e^{-x}\,dx = \sqrt{(n+1)} = n!$, n is an integer. [7 marks]

(ii) Evaluate $\int_0^{\infty} \frac{e^{-x}}{1+x^2} dx$ using the formulae in (i) above. [4 marks]

(b) Apply the Euler-Cauchy method with step length h to the problem

$y' = -y, y(0) = 1$

The Euler-Cauchy method is given by

$k_1 = h f(x_n, y_n)$

$k_2 = h f(x_n + h, y_n + k_1)$

$y_{n+1} = y_n + ½ (k_1 + k_2)$

(i) Determine an explicit expression for y_n. [5 marks]

(ii) For which values of h is the sequence $\{y_n\}_0^{\infty}$ bounded? [2 marks]

(iii) Compute $\lim_{h \to 0} \{(y(x; h) - e^{-x})/h^2\}$ [5 marks]

Question 3

(a) Using Crank-Nicolson method, solve the following differential equation

$\frac{\partial^2 y}{\partial x^2} - 16\frac{\partial y}{\partial t} = 0,\ 0 < x < 1,\ \ t > 0$ given u(x, 0), u(0,t) = 0, u(1, t) = 50t

Compute u for two steps in t –direction taking h = ¼ [12 marks]

(b) Compute an approximation to y(1), y'(1) and y''(1) with Taylor's algorithm of order two and step length h = 1 when y(x) is the solution to the IVP

$y''' + 2y'' + y' - y = \cos x, 0 \leq x \leq 1, y(o) = 0,$

$y'(o) = 1, y''(o) = 2$ [7 marks]

(c) Given the equation y' = x + siny with y (o) = 1, show that it is sufficient to use Euler's method with the step h = 0.2 to compute y (0.2) with one error les then 0.05. [4 marks]

Question 4

(a) Solve the following wave equation

$\frac{\partial 2y}{\partial t^2} = 4\ \frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}}$ with the boundary conditions u(0, t) = u(4, t) = 0 and initial-condition

$\frac{\partial y}{\partial t}$ (x, 0) = 0, u(x, 0) = x (4 − x). Take h = 1.0. [10 marks]

(b) Consider the following Range-Kutta method for the differential equation y' = f(x,y)

$W_i + 1 = W_i + \frac{1}{6}(k_1 + 4k_2 + k_3)$

$k_1 = h\,f(x_i, w_i)$, $k_2 = h\,f\ x_i + h/2, w_i + k_{1/2}$

$k_3 = h\,f\left(x_i + h, w_i - k_1 + 2k_2\right)$

(i) Compute y(0.4) when $y' = \frac{y+x}{y-x}$, y (o) = 1 and h = 0.2. Round to five decimals.

[8 marks]

(ii) What is the result after one step of length h when y' = y, y (o) = 1? [5 marks]

Question 5

(a) Solve the equation

$$\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 10\left(x^2 + y^2 + 10\right) = 0$$

Over the square mesh with sides x = 0, y = 0, x = 3. y = 3 with n = 0 on the boundary and mesh length = 1. [10 marks]

(b) Derive the 2-point (n = 1) Gauss-Hermite formula and calculate the error function. Hence or otherwise evaluate

$I = \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x^2 + x - 1}\,dx$ using the Gauss-Hermite 2-point formulas. [9 marks]

(c) Evaluate the integral

$$\int_1^2 \int_1^2 \frac{dx\,dy}{x + y}$$

Using the trapezoidal rule with h = k = 0.5 [4 marks]

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