KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 537: METHODS OF APPLIED MATHEMATICS

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DATE: TUESDAY 1ST FEBRUARY 2011 TIME: 9.00 A.M. - 12.00 NOON

INSTRUCTIONS

Answer any THREE questions.

Question One (25 marks)

a) Differentiate between

i) a Fredholm equation of first kind and a Fredholm equation of second kind. (2 marks)

ii) a Fredholm equation and a Voltera equation. (2 marks)

b) Solve the following integral equations

i) $u(x)=\lambda\int_{o}^{\frac{\pi}{2}} \sin x \sin y \ u(y) \ dy$ (8 marks)

ii) $u(x) = \cos h(x) - \frac{x}{2} + \frac{1}{3}\int_{o}^{1} xy \ u(y) \ dy$ (11 marks)

Question Two (23 marks)

a) Obtain the Fourier series for $f(x) = x^2$ when $-\pi < x < \pi$. Hence prove that

$$\frac{1}{12} + \frac{1}{2^2} + \frac{1}{3^2} + ... = \frac{\pi^2}{6}$$ (12 marks)

b) Find the Fourier expansion of the periodic function whose definition in one period is

$$f(t) = \begin{cases} 0 & -\pi < t < 0 \\ \sin t & 0 < t < \pi \end{cases}$$ (11 marks)

Question Three (23 marks)

a) Let $g(t) = \begin{cases} e^t & for \quad 0 \leq t \\ -1 + e^t + e^{t-1} & for \ 1 \leq t \end{cases}$

i) Show that g(t) is continuous for $t \geq 0$ but $g^1(t)$ is not defined. Also show that $g\ (t)$ is a solution of $y^1 - y = H(t-1), = y\ (0) = 1$ for $t \geq 0, \quad t \neq 1.$

(6 marks)

ii) Show that g(t) satisfies the integral equation

$y(t) = \int_o^t y\ (\tau) + H\ (\tau - 1)\ d\,\tau + 1$ for all $t \geq 0$ including $t = 1.$

(Note : H (t-1) is the heaviside step function) (6 marks)

b) Suppose that L(t) and h(t) are continuously differentiable and h(0) = 0,

(i) If $u(t)$ is a continuous solution of $\int_o^t L\ (t-r)\ u(r)\ dr = h(t)$, show that $L(0)\ u(t) + \int_o^t L^1\ (t-r)\ u(r)\ d\,r = h^1\ (t).$ (4 marks)

ii) Show that if a continuous function h(t) satisfies h(0) = 0 and u(t) solves $L(0)\ u(t) + \int_0^t L^1\ (t-r)\ u(r)\,dr = h^1(t)$ then it must solve

$\int_o^t L\ (t-r)\ u(r)\,dr = h(t)$ (7 marks)

Question Four (23 marks)

a) The following spaces are defined as:

$$M_1 = \{\phi(x) \,;\, \phi \in C^2 \,[0, 1] \ \ and \ \ \phi\,(0) = 0\}$$

$$M_2 = \{\phi\,(x) \in M_1 : \phi(1) = 0\,\}$$

i) Let J(v) $= \int_o^1 \left(k(v^1)^2 - 2fv)\right) dx,, \ \ V \in M_1, \quad K \in C^1 \ (0,1)$

and $f \in C[0\,,1]$. Explain what is meant by saying $u \in M_1$ is a minimizer of J on M_1. (4 marks)

ii) Show that $u \in M_1$ is a mini of J on M_1 if and only if u satisfies

$\int_o^1 (K u^1 \ \phi^1 - f\,\phi\,)\,d\,x = 0$; for every $\phi \in M_1$. (9 marks)

b) With ϕ *and* M defined in (a) , let $h(x) \in C[0,1]$ and $\int_o^1 h(x)\phi(x)\,dx = 0$ for all $\phi \in M_2$, show that h(x) is identically equal to zero. (10 marks)

Question Five (23 marks)

a) Define the Green's function as used in differential equations. (3 marks)

b) Find the Green's function for the following boundary value problems and hence solve them or find the expression for their solutions.

i) $\dfrac{d^2 y}{dx^2} = -\,f(x)$

$y(0) = 0$

$y(1) + y^1(1) = 2$ (10 marks)

ii) $\dfrac{d^2 y}{dx^2} + u = -\,f\,(x) \qquad 0 < x$

$u(0) = u^1 \ (0) = 0$ (10 marks)