KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 562: MULTIVARIATE ANALYSIS I

DATE: Monday 28th November, 2011 **TIME:** 9.00 a.m. – 12.00 p.m.

INSTRUCTIONS

Answer Question 1 and ANY OTHER TWO questions.

Question 1

(a) Let $\underline{x}$ be a 3-variate random vector such that

$$\text{Var}\left(\underline{x}\right)=\begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix}=\Sigma$$

Determine the variance-covariance matrix of the random vector $\underline{Y} = (Y_1, Y_2, Y_3)'$

Where

$Y_1 = x_1 + x_2$

$Y_2 = x_1 + x_3$

$Y_3 = x_2 + x_3$ [8 marks]

(b) Let $\underline{x}$ be a p-variate random vector with mean vector $\underset{\sim}{\mu}$ and variance-covariance matrix

$$\Sigma = \left(\left(\sigma_{i\,j}\right), i, J = 1,2,\ldots, p\right)$$

(i) Show that $\mathrm{E}\left(x!\ A\ x\right) = \mathrm{Trace}\left(A\Sigma\right) + \underset{\sim}{\mu}'\ A\ \underset{\sim}{\mu}$ [7 marks]

(ii) If p = 3 and Σ is as in part (a), then find the expected value of the quadratic form

$Q = \left(x_1 - x_2\right)^2 + \left(x_2 - x_3\right)^2 + \left(x_3 - x_1\right)^2$ [7 marks]

(c) Let $x_1, x_2, \ldots, x_p$ have the joint multinomial distribution given by

$$f\left(x_1, x_2, \ldots x_p\right) = \frac{n!}{x_1!\,x_2!\ldots x_p!}\, p_1^{x_1}\ p_2^{x_2} \ldots\ldots p_p^{x_p}\ p_{p+1}^{x_{p+1}}$$

Where $\sum_{i=1}^{p} x_i = n, \quad 0 < p_i < 1, \quad i = 1, 2, ..., p,$

$$p_{p+1} = 1 - \sum_{i=1}^{p} p_i, \quad x_{p+1} = n - \sum_{i=1}^{p} x_i$$

Find the moment generating function of the distribution, hence find $E(x_i)$ and $Var(x_i)$.

[8 marks]

Question 2

(a) Let $\underline{x}_i$, i = 1, 2, …, n be iid p-variate random vectors with mean $\underset{\sim}{\mu}$ and dispersion matrix Σ. The maximum likelihood estimators of $\underset{\sim}{\mu}$ and Σ are $\bar{\underline{x}}$ and s respectively.

(i) Write down typical elements of the vector $\bar{\underline{x}}$ and the matrix s. [2 marks]

(ii) Give the expression for the hotelling T^2 statistic. [2 marks]

(b) Derive the likelihood ratio test (LRT) of size α based on the Hotelling T^2- statistic for

$H_o : \underset{\sim}{\mu} = \underset{\sim}{\mu}_o$ against $H_1 : \underset{\sim}{\mu} \neq \underset{\sim}{\mu}_o$

($\underset{\sim}{\mu}_o$ is a specified vector of constants).

Show that the test statistic is related to Fisher's F distribution with degrees of freedom $r_1 = p$ and $r_2 = n - p$ [16 marks]

Question 3

Let $\underline{x}_i$, i = 1, 2, …, n be independent p-variate vectors identically distributed as Np ($\underline{O}$, Σ)

Further let A = $B + \underset{\sim}{x}_1 \underset{\sim}{x}_1'$

$Y = A^{-½} \underline{x}_1$

Where B = $\sum_{i=2}^{n} \underline{x}_i \underset{\sim}{x}_i'$

(a) State the distributions of A and B respectively. [4 marks]

(b) Show that the joint density of A and $\underline{y}$ is given by

$$g(A, \underline{y}) = c_1\,(n, p)\,\mathrm{et}\gamma - \tfrac{1}{2}\left(\Sigma^{-1}\,A\right).|A|^{\frac{n-p-1}{2}}\,\left(1 - \underline{y}'\,\underline{y}\right)^{\frac{n-p-2}{2}}$$

where for $\underline{y}'\ y < 1$

$$c_1(n,p) = \left\{\pi^{p/2}\ \Gamma_p\left(\frac{n-1}{2}\right)|2|^{n/2}\right\}^{-1}$$

$$\Gamma_p\left(\frac{r}{2}\right) = \prod_{i=1}^{p}\left\{\left[T\left(\tfrac{1}{2}\right)\right]^{i-1}\,\Gamma\left(\frac{\gamma - i + 1}{20}\right)\right\}$$

Hence show that the marginal distribution of $\underline{y}$ is

$\lambda(\underline{y}) = c_2\,(n, p)\left(1 - \underline{y}'\,\underline{y}\right)^{\frac{n-p-2}{2}}\ ,\ \underline{y}'\,\underline{y} < 1$ [16 marks]

Question 4

(a) Prove that the random vector $\underline{x}$ will have the p-variate normal distribution if and only if the random variable

$y = \underline{a}'\,\underline{x}\ \left(\underline{a} \neq \underline{o}\right)$ has a univariable normal distribution. [5 marks]

(b) Let $\underline{x}$ and $\underline{y}$ be p x 1 and q x 1 random vectors respectively such that

$\underline{x} \sim N(\underline{\mu}_1, \Sigma_{11})$, $\underline{y} \sim N(\underline{\mu}_2, \Sigma_{22})$

cor $(\underline{x}, \underline{y}) = \Sigma_{12}$, cor $(\underline{y}, \underline{x}) = \Sigma_{21}$

Based on a random sample $\left(\underline{x}_{i,}\ \underline{y}_i\right)$

i = 1, 2, …, n. Use the union-intersection principle to derive a test for stochastic independence of $\underline{x}$ and $\underline{y}$. [15 marks]

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