KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 562: MULTIVARIATE ANALYSIS I

DATE: Wednesday 28th March, 2012 **TIME:** 9.00 a.m. – 12.00 p.m.

INSTRUCTIONS

Answer Question One and any other TWO questions.

Q1. **COMPULSORY (30 MARKS)**

(a) Define a random vector.

(b) Consider the bivariate Normal distribution with $\mu_1 = 1,\ \mu_2 = 3, \sigma_{11} = 2\ \ \sigma_{22} = 1\ \ \rho_{12} = -0.8$

Write down the bivariate Normal Distribution.

(c) Define the Quadratic form Q(x) = $\left(\underset{\sim}{x} - \underset{\sim}{\mu}\right)^1 \Sigma^{-1}\left(\underset{\sim}{x} - \underset{\sim}{\mu}\right)$ Show that Q(x) ~ χ^2_p.

(d) Given

$$F(x, y) = \begin{cases} 0 & x \le 0 \quad y \le 0 \qquad x + y \le 1 \\ 1 & \text{elsewhere} \end{cases}$$

Determine whether F(xy) is a cumulative distribution function.

(e) Let $\underset{\sim}{x} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$ with $\underset{\sim}{\mu}^1 = [-3, 1, 4]$

and $\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Determine which of the following are independent random variables

(i) X_1 and X_2

(ii) $\frac{1}{2}\left(X_1 + X_2\right)$ and X_2

(f) Define a characteristic function of random variable. If $X \sim N_p(\underset{\sim}{\mu}\,\Sigma)$ show using characteristics function that $\underset{\sim}{y} = \left(\underset{\sim}{x} - \underset{\sim}{\mu}\right)\Sigma^{-1} \sim N_p(01)$

Q2. (a) Let $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}\,\Sigma)$

Prove that $\underset{\sim}{x}_1 / \underset{\sim}{x}_2 \sim N_2\left(\underset{\sim}{\mu}_{1.2}, \Sigma_{11.2}\right)$

where $\underset{\sim}{\mu}_{1.2} = \underset{\sim}{\mu}_1 + \Sigma_{12}\,\Sigma_{22}^{-1}\left(\underset{\sim}{x}_2 - \underset{\sim}{\mu}_2\right)$

and $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12}\,\Sigma_{22}^{-1}\,\Sigma_{21}$

(b) Given $X \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$ with $\underset{\sim}{\mu} = \underset{\sim}{o}$

and $\Sigma = \begin{bmatrix} 1.0 & 0.6 & -0.3 \\ 0.6 & 1.0 & -0.25 \\ -0.3 & -0.25 & 1 \end{bmatrix}$

(i) Find the conditional distribution of X_1 and X_3 given X_2.

(ii) Partial correlation between X_1 and X_3 given X_2.

(iii) Multiple correlation coefficient between X_1 and (X_2, X_3)

Q3. (a) Define the central wishart distribution.

(b) If $\bar{y} = n^{-1}\sum_{i=1}^{n} y_i,\ S = (n-1)^{-1}\sum_{i=1}^{n}(y_i - \bar{y})(y_i - \bar{y})^1$

where y_i are independent and identically distributed $N_p\left(\underset{\sim}{\mu}\,\Sigma\right)$ show that

(i) $\underset{\sim}{\bar{y}} \sim N\left(\underset{\sim}{\mu}, \Sigma/n\right)$

(ii) $(n - 1)\,S \sim w\,(\Sigma, \rho, n - 1)$

(iii) $\bar{y}$ and S are independent

(iv) $\bar{y}$ and s are unbiased estimators of $\underset{\sim}{\mu}$ and Σ respectively.

Q4. (a) Let H_o: $\underset{\sim}{\mu} = K\,\underset{\sim}{\mu}\,o$

H_1: $\underset{\sim}{\mu} \neq \underset{\sim}{\mu}_o$

Where Σ is unknown. Obtain the Maximum Likelihood Estimator of $\underset{\sim}{\mu}$.

(b) Derive the likelihood ratio test for testing the hypothesis

H_o : $\underset{\sim}{\mu} = \underset{\sim}{\mu}_o$ against

H1: $\mu \neq \underset{\sim}{\mu}_o$

(c) Given that

$$\bar{x} = \begin{bmatrix} 55 \\ 40 \end{bmatrix} \text{ and } S = \begin{bmatrix} 210 & 127 \\ 127 & 120 \end{bmatrix}$$

Test the hypothesis that

$H_o : \mu = \mu_o$ where $\mu_o = \begin{bmatrix} 52 \\ 42 \end{bmatrix}$ against

H_1: $\mu \neq \mu_o$

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