KENYATTA UNIVERSITY

UNIVESITY EXAMINATIONS 2011/2012
SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING

EEE 314: COMPLEX ANALYSIS FOR ELECTRICAL ENGINEERS

DATE: **Thursday 19th April 2012** **TIME: 8.00a.m – 10.00a.m**

INSTRUCTIONS: Answer question ONE and any other two

Question One (30 marks)

a). Find square roots of $-3+4i$. (4 marks)

b). Obtain the general solution of $\ln\left(-\sqrt{2}+i\sqrt{2}\right)$ and hence state the principal value. (4 marks)

c). Obtain the Laurent's series expansion of $f(z)=\dfrac{3z-3}{z^2-z-2}$ about the singular point $z=-1$. Use the series obtained to find the residue of $f(z)$ at $z=-1$ and hence evaluate $\int_C \dfrac{3z-3}{z^2-z-2}dz$ where $C:|z|=1$. (6 marks)

d). Use Cauchy's theorem to evaluate the following integral $\int_c \dfrac{1-2z}{z(z-1)(z-3)}dz$ Where $C:|z|=2$. (6 marks)

e). Use Cauchy's integral theorem to evaluate $\int_C \dfrac{5z^2+2z+1}{(z-i)^3}dz$, Where $C:|z+i|=2$ (4 marks)

f). Let $v(x,y)=3x^2y-y^3-x$. Find $u(x,y)$ such that $f(z)=u(x,y)+iv(x,y)$ is harmonic. Express $f(z)$ in terms of z. (6 marks)

Question two (20 marks)

a) Show that $\sin^{-1}(z)=-i\ln\left(iz+\sqrt{1-z^2}\right)$. Hence find all solutions to the equation $\sin z=3i$ (7 marks)

b) Evaluate $\lim\limits_{z\to 1+i}\left(\dfrac{z^2-z+1-i}{z^2-2z+2}\right)$ (4 marks)

c) If $z=3-i$, evaluate $f(3-i)$ where $f(z)=z^3-2z^2-(4+5i)$ (3 marks

d) Show that $|\sin z|^2=\sin^2 x+\sinh^2 y$ (6 marks)

Question three (20 marks)

a) Show that a necessary condition for a function $f(z) = u(x,y) + iv(x,y)$ to be analytic in a region R is that the four partial derivatives $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}$ exist and satisfy Cauchy Riemann conditions. (7 marks)

b) Show that $u(x,y) = \frac{1}{2}\ln(x^2 + y^2)$ is harmonic. Find $v(x,y)$ such that $u(x,y) + iv(x,y)$ is harmonic. Express $f(z)$ in terms of z. (8 marks)

c) Determine whether $f(z) = e^{y}\cos x + ie^{y}\sin x$ is analytic. Justify your answer (5 marks)

Question Four (20 marks)

a) Show that if a function $f(z)$ is analytic inside and on a simple closed contour C, then $\int_C f(z)dz = 0$ (7 marks)

b) Find all values of $(1+i)^{1+i}$. (4 marks)

c) Solve the equation $\ln(z^2 - 1) = \frac{i\pi}{2}$. (5 marks)

d) Use De-Moivres theorem to show that $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$ (4 marks)

Question Five (20 marks)

a) If $f(z)$ is analytic inside and on a simple closed curve C, and if z_0 is inside C, then $f'(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^2}dz$ .Show that the second derivative of $f(z)$ at z_0 is given by

$$f''(z_0) = \frac{2!}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^3}dz.$$ (8 marks)

b) Evaluate $\int_{3i}^{2+4i}(2y + x^2)dx + (3x - y)dy$ along the curve $x = 2t$, $y = 3 + t^2$ (6 marks)

c) State without proof Cauchy's residue theorem. Hence use it to solve the following integral

$$\int_C \frac{z^2+4}{z^3+2z^2+2z}dz \text{ where } C:|z| = 3$$ (6 marks)