KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE (STATISTICS)

SMA 564: SAMPLE SURVEY

DATE: **Friday 30th March 2012** **TIME:** **9.00a.m – 12.00p.m**

INSRTUCTION: Answer Question One and Any Other Two Questions

Question 1 (30 Marks)

(a) Differentiate between proportional allocation and optimum allocation of sample size in stratified random sampling. (4 Marks)

(b) Prove that in proportional allocation, $\text{var}(\bar{y})_{prop} = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{i=1}^{k} W_i S_i^2$ (6 Marks)

(c) Let a random variable $Y_i = \begin{cases} E(Y_i) = \beta x_i \\ Var(Y_i) = \sigma^2 (x_i^2) \\ Cov(Y_i, Y_j) = 0, i \neq j \end{cases}$

where x_i is an auxiliary variable corresponding to the ith survey variable. Show that the Horvitz-Thompson estimator $\bar{y}_{HT}$ is unbiased under the above model (7 Marks)

(d) Explain the concept of ***robustness problem*** in sample surveys (5 Marks)

(e) Prove that the error variance of the ratio estimator under a simple regression model

$$Var_\xi\left(\bar{y}_R - \bar{Y}\right) = \frac{\sigma^2}{N^2} \frac{\sum_{i \in s} x_i}{\sum_{i \in s} x_i} \sum_{1}^{N} x_i$$ (8 Marks)

Question 2 (20 Marks)

(a) Suppose $y_1, y_2, y_3, \ldots y_n$ is random sample of size n from a population of size N drawn by simple random sampling without replacement. Let b_i ={1 if the ith unit is in the sample, and 0 otherwise. Show that

(i) the i^{th} sample mean is unbiased estimator of the population mean. (6 Marks)

(ii) the variance of the sample mean as an estimator of the population mean is given by

$$\text{Var}\ \bar{y} = \frac{N-n}{Nn} s^2$$ (7Marks)

(b) Let $y_1, y_2, y_3, \ldots y_N$ be a population consisting of N units. Suppose the population consists of K strata, the ith stratum consists of Ni units such that $\sum_{i=1}^{K} N_i = N$. After a stratum has been determined a simple random sample of size n_i from the ith stratum is chosen such that $n_i = \sum_{i=1}^{K} n_i$.

Determine the optimal value of n_i when the cost function is

$$c = a + \sum_{i=1}^{K} c_i n_i$$, where c is some constant (7 marks)

Question 3 (20 Marks)

(a) Let π_i be the probability of including unit i in the sample. Define α_i ={1, if the ith unit is in the sample, and 0 otherwise.

Further, let Y and X be the survey variable and auxiliary variable respectively,

prove that the Horvitz-Thompson estimator $\bar{y}_{HT} = \sum_{i \in s} \frac{y_i}{N\pi_i}$ is unbiased for $\bar{Y}$ (5 Marks)

(b) (i) Derive ~~the simplification of~~ the simplification for the variance of the Horvitz –Thompson, $Var\left(\bar{y}_{HT}\right)$ due to Yates and Grundy (9 Marks)

(ii) Explain a drawback for the variance estimator due to Yates and Grundy (1 Mark)

(c) Derive the error variance of the Horvitz-Thompson estimator $\bar{y}_{HT}$ (5 Marks)

Question 4 (20 Marks)

Let the population consist of N clusters of M elements each. Suppose n clusters are selected from N clusters by simple random sampling without replacement,

Define Y_{ij}=the value of the characteristic under study for the jth element of the ith cluster

$\bar{Y}_i = \frac{1}{M}\sum_{j=1}^{M} Y_{ij}$ =the mean per element of the ith cluster

$\bar{Y} = \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M} Y_{ij}$ =the population mean

$S_i^2 = \frac{1}{M-1}\sum_{j=1}^{M}\left(Y_{ij} - \bar{Y}_i\right)^2$ the mean square between elements of the ith cluster

$S_b^2 = \frac{1}{N-1}\sum_{j=1}^{M}\left(Y_i - \bar{Y}\right)^2$ =the mean square between cluster mean

Suppose the clusters are equal in size.

(i) Determine an estimator of $\bar{Y}$ (5 Marks)

(ii)Derive the variance estimator of $\bar{Y}$ (15 Marks)