KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 565: THEORY OF ESTIMATION

DATE: Thursday 1st December 2011 TIME: 9.00a.m -12.00p.m

INSTRUCTIONS:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

Question 1

a) Let $x_1, x_2, \ldots x_n$ be a random sample from the family of distributions $\{f(x;\theta), \theta \in \Omega\}$ and T = ($T_1, T_2, \ldots T_x$) be a set of statistics. Explain the following concepts as used in statistical Theory of Estimation [6marks]

i) Joint sufficiency

ii) Minimal sufficiency

iii) Completeness

b) Distinguish between the following: [6marks]

i) Locally Minimum Variance Unbiased Estimator (LMVE) and Uniformly Minimum Variance Unbiased Estimator (UMVUE)

ii) Minimax and Bayesian Estimate

iii) Minimum Variance Bound Unbiased (MVBUE) and UMVUE. State which implies the other.

c) Let x_1 x_2 …, x_n be iid random variables from the pdf

$f(x;\alpha) = e^{-x/\alpha,} \quad , x > 0$

Zero otherwise

where $\alpha\ > 0$ is unknown parameter, Find

i) The maximum Likelihood Estimator (MLE) of $\alpha$and show that it is unbiased for α. [6marks]

ii) The UMVUE of α [6marks]

iii) The MVBUE of α if it exists [6marks]

Question 2

a) $Let\ F(x;\theta\ ), \theta \in \Omega\}$ be a family of distributions and h be statistic in the set $U = \{h\ ; E\ (h) =\ \theta,\ E(h^2)\ < \infty\ for\ all\ \theta \in \Omega\}$ let T be a sufficient statistic for the above family. Then show that,

i) E(h/T) is a statistic and is unbiased for θ

ii) If $\emptyset(T) = E(h/T) then$

Var $\{\emptyset(T)\} \leq$ Var (h) for all $\theta \epsilon \Omega$

[9marks]

b) Let x_1, x_2, … x_n be iid with the probability function

f(x;θ)= $e^{-\theta}\ \frac{\theta^x}{x!}\ ,\ x = 0,1,2, --$

zero otherwise

In general Kolmogorov's theorem asserts that if T is a sufficient statistic for θ then the conditional distribution of x given T is the UMVUE of $f(x;\theta)$. Use this result to obtain the UMVUE of $f(x;\theta)$ as given above. [11marks]

Question 3

Let x_1, x_2,…x_n be iid N$(\theta, 1)$ random variables.

a) Find the method of moment estimator of θ^2 and give its large sample distribution. [8marks]

b) Find the maximum likelihood estimator (MLE) of $Pr\ \{X > c\}$ where c is a constant and determine the large sample distribution of the MLE [12marks]

Question 4

a) Let Ω be an open interval of the real line and $\{ f(x;\theta),\ \theta \varepsilon \Omega\}$ be a family of pdf's (pmf's). Assume that the support of $f(x;\theta)$ is independent of θ. Given $\varphi(\theta)$ and $\emptyset(\theta)$ are defined on Ω and T(x) is defined on R_n such that T(x) is unbiased for $\varphi(\theta)$. State clearly the regularity conditions for the Cramer'-Rao lower bound for the variance of unbiased estimator. Hence state and prove the Cramer'-Rao inequality. [10marks]

Let $x_1, x_2, \text{---}\ x_n$ be iid with the pmf

$f(x;\lambda) = e^{-\lambda} \frac{\lambda^x}{x!}, \qquad x = 0,1,2,--$

zero otherwise

Show that there does not exist an unbiased estimator of λ^2 whose variance attains the cramer'-Rao lower bound [10marks]

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