KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2009/2010

INSTITUTIONAL BASED PROGRAMME (AUGUST SESSION)

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 566: HYPOTHESIS TESTING

DATE: THURSDAY 30TH DECEMBER 2010 TIME: 9.00 A.M. - 12.00 NOON

INSTRUCTIONS

Answer Any THREE questions.

Question 1

a) Define the following

i) a simple hypothesis

ii) a Most Powerful test (2 marks)

b) Let $\{ f(x);\theta) \; \theta \in \Omega\}$ be a family of distributions with $\Omega = \{\theta_o, \theta_1\}$. Give the form of the Most Powerful (M P) size α test for testing $H_o : \theta = \theta_o$ against $H_1 : \theta = \theta_1$ and prove that it is Most Powerful of its size. (10 marks)

c) Let X_1 --, X_n be a sample of size n from the pmf

$$f(x : \theta) = e^{-\theta} \frac{\theta^x}{x!} \;' \; x = 0, 1, 2, ---$$

Find a Most Powerful (MP) size α test for $H_o : \theta = \theta_o$ against $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$. (11 marks)

Question 2

a) Distinguish between

i) "Randomized Test" and "Non-Randomized Test". (4 marks)

ii) "Most Powerful Test" and "Uniformly Most Powerful Test". (3 marks)

b) Let $X_1, X_2, \ldots, X_n$ be *iid* random variables with the gamma pdf given by

$$f(x;\theta) = \frac{x^{p-1}}{\theta^p \Gamma(p)} \exp\left\{\frac{-x}{\theta}\right\}, \quad x > 0$$

where $p \in \mathbb{N}$ is known and $\theta > 0$. Find a UMP size α test for testing the null hypothesis $H_o : \theta_1 \leq \theta \leq \theta_2$ against the alternative hypothesis

$H_1 : \theta < \theta_1 \;\; or \;\; \theta > \theta_2,$ where $0 < \theta_1 < \theta_2 < \infty$. (6 marks)

c) Let $X_1, X_2, \ldots, X_n$ be *iid* random variables with the pdf

$f(x:\lambda) = \lambda \; e^{-\lambda x}, x > 0$

where $\lambda > 0$ is unknown parameter. Determine the Uniformly Most Powerful (UMP) size α test for $H_o : \lambda \geq \lambda_o$ against $H_1 : \lambda < \lambda_o$, where λ_o is a specified value of λ. Hence give, in terms of a statistic in which $f(x:\lambda)$ has an MLR, the UMP size α test. (10 marks)

Question 3

a) Define the following terms

i) Unbiased test

ii) Uniformly Most Powerful unbiased test.

iii) Consistent test (6 marks)

b) Explain the concept of "Monotone Likelihood Ratio" as refers to a family of probability density (or probability mass) functions. (3 marks)

c) Let Let X_1 , X_2 ,... , X_n be *iid* random variables with probability mass function (pmf)

$$f(x\,;\theta) \;=\; e^{-\theta}\,\frac{\theta^{x}}{x\,!}\;,\quad x = \;0,\,1,\,2,\,....\qquad (0<\theta<\infty)$$

Find the UMP unbiased size α test for testing the null hypothesis $H_o:\theta=\theta_o$ against the alternative hypothesis $H_1\,:\,\theta\neq\theta_o$ where θ_o is a specified number. Give an expression for the power function. (14 marks)

Question 4

a) State the Likelihood Ratio (LRT) principle. (3 marks)

b) Show that if the family of pdf's (pmf's) $\{f(x\,;\theta)\,,\;\;\theta\in\,\Omega\}$ admits a sufficient statistic T(x) then for testing $H_o:\theta\in\Omega$ against $H_1:\theta\in\Omega-\Omega_o$,

the LRT is a function of the sufficient statistic. (3 marks)

c) Let X_1 , X_2, ..., X_n be a random sample from the pdf.

$$F(X{:}\theta)=\begin{cases}\dfrac{1}{\sqrt{2_\pi}}\ \exp\ \{-\tfrac{1}{2}\ (x-\theta^2)\}\ , & -\infty<x<\infty\\ \quad 0\,, & otherwise\end{cases}$$

Where $\infty\in(-\infty\,,\infty)$ is the mean of the distribution. Determine the Likelihood Ratio Test (LRT) of size or for testing $H_o:\theta=\theta_o$ against $H_1\,:\,\theta\neq\theta_o$

Give a large sample test for H_o against H_1. (17 marks)

Question 5

a) Let $X_1, X_2, \ldots, X_n$ be *iid* $N(\mu, \sigma^2)$. Derive the Most Powerful (MP) size α test for testing

$$H_0 : \mu = 2 \ , \sigma = 1$$

against $\quad H_1 : \mu = 3 \ , \sigma = 2$

and show that the Best Critical Region (BCR) is of the form

$$C = \{x : \sum_{i=1}^{n} (x_i - \frac{5}{3})^2 > k^*\}$$

where $\quad k^* > 0$.

b) Let $X_1, X_2, \ldots, X_n$ be *iid* from the pdf

$$f(x\,;\,\theta) = \begin{cases} \dfrac{1}{\theta}\ , & 0 < x < \theta \\ 0\ , & otherwise \end{cases}$$

Derive a size α Likelihood Ratio Test (LRT) for the hypothesis.

$H_o : \theta = \theta_o$ against $H_1 : \theta \neq \theta_o$

Show the test derived is unbiased. (12 marks)