KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

INSTITUTIONAL BASED PROGRAMME (AUGUST SESSION)

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 567: STOCHASTIC PROCESSES

DATE: THURSDAY, 28TH APRIL 2011 **TIME: 9.00 A.M. - 12.00 P.M.**

INSTRUCTIONS: Answer any THREE questions.

1. Write down the differential difference equations for the linear growth process with immigration.

   $\lambda_n = n\lambda + a, \;\; \mu_n = n\mu.$ (2 marks)

   Show that

   $M(t) = E[Z(t)]$

   Satisfies the differential equation

   $M(t) = (\lambda - \mu)\, M(t) + a.$

   (8 marks)

   Hence show that with the initial condition

   $M(0) = i, \;\; if \;\; Z(0) = i,$ then

   $$M(t) = \frac{a}{\lambda - \mu}\left\{e^{(\lambda-\mu)t} - 1\right\} + ie^{(\lambda-\mu)t}, \lambda \neq \mu$$ (8 marks)

   and taking limit as $\lambda \to \mu,$ show that $M(t) = at + i$ for $\lambda = \mu$. (3 marks)

   What is the limit of $M(t)$ as $t \to \infty$ for $\lambda < \mu$? (2 marks)

2. If S_1, S_2, S_3, S_4 are four points in a circle, and a boy steps from one step to the other according to the following transitional probability matrix.

$$\begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{5} & 0 & \frac{4}{5} & 0 \\ 0 & \frac{1}{5} & 0 & \frac{4}{5} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(a) Determine:

(i) the fundamental matrix, (4 marks)

(ii) the matrix for the variance of the number of times that an absorbing chain is in a transient state, (4 marks)

(iii) the matrix for the probability that an absorbing chain arrives in an absorbing state given that it began in a transient state. (4 marks)

(b) If the starting point is S_2, what are

(i) the mean and variance of the number of times the boy is at point S_3, (4 marks)

(ii) the mean and variance of the number of steps the boy takes before arriving in S_1 or S_4, (4 marks)

(iii) the probability that the boy is absorbed in S_4? (3 marks)

3. (a) Give a summary of how to obtain an nth step transition probability matrix by the algebraic treatment of finite markov chains. (8 marks)

(b) Obtain the nth step transition probabilities of the following transition matrix:

$$\begin{bmatrix} 1-\beta & \beta \\ \alpha & 1-\alpha \end{bmatrix}$$

where $0 < \alpha < 1$ and $0 < \beta < 1$ (15 marks)

4. (a) The process $\{X(t), t \in T\}$ has a distribution

$$\Pr.\{X(t)=n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}} & , \quad n = 1,2,3, \_\_\_ \\ \dfrac{at}{1+at} & , \quad n = 0 \end{cases}$$

Show that this process is not stationary. (8 marks)

(b) Classify the states of the following transitional matrix

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & - & - & - \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & - & - & - \\ \frac{3}{4} & 0 & 0 & \frac{1}{4} & 0 & 0 & - & - & - \\ \frac{4}{5} & 0 & 0 & 0 & \frac{1}{5} & 0 & - & - & - \\ . & . & . & . & . & . & - & - & - \\ . & . & . & . & . & . & - & - & - \\ . & . & . & . & . & . & - & - & - \end{bmatrix}$$

(15 marks)

5. (a) Using generating functions show that in the Branching process

$$Var(Z_n) = \begin{cases} \sigma^2 \mu^{n-1} \dfrac{(1-\mu^n)}{1-\mu} & , \quad \mu \neq 1 \\ n\sigma^2 \quad for & \quad \mu = 1 \end{cases}$$

(10 marks)

(b) Show that the bivariate generating function of X_m and X_n is $G_m[s_1 G_{n-m}(s_2)]$. Hence or otherwise show that

$E(X_n X_m) = \mu^{n-m} E(X_m{}^2)$ if $n > m$ and $\mu = E(X)$ (13 marks)

****