KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE

SMA 572: INTRODUCTION TO PROBABILITY THEORY

DATE: **Tuesday 3rd April 21012** **TIME:** **2.00p.m – 5.00p.m**

INSTRUCTIONS

Answer question ONE and any other TWO questions.

QUESTION ONE (30 marks)

(a) Events $A \text{ and } B$ are such that

$$P(A/B)=0.4,\ P(B/A)=0.25 \text{ and } P(A\cap B)=0.12$$

(i) Calculate the value of $P(B)$. (1 mark)

(ii) Give a reason why $A \text{ and } B$ are not independent. (1 mark)

(iii) Calculate the value of $P(A\cap \overline{B})$. (1 mark)

(b) A doctor assesses the performance of 50 nurses. It is found that 14 regularly complete their contracts behind schedule (A), 15 produce poor quality work (B) and 6 fall into both categories (A and B). A nurse is randomly selected. Find the probability that:

(i) She will be behind schedule. (1 mark)

(ii) She will be called back to remedy poor work. (1 mark)

(iii) She will be unsatisfactory (behind schedule or poor quality). (1 mark)

(c) In a certain selection of flower seeds $\frac{2}{3}$ have been treated to improve germination and $\frac{1}{3}$ have been left untreated. The seeds which have been treated have a probability of germination of 0.8, whereas the untreated seeds have a probability of germination of 0.5.

(i) Find the probability that a seed selected at random will germinate.

(2 marks)

(ii) If the seeds were sown and given time to germinate, find the probability that a seed selected at random had been treated given that it had germinated. (2 marks)

(d) A computer program generates random questions in arithmetic that children have to answer within a fixed time. The probability of the first question being answered correctly is 0.8. Whenever a question is answered correctly, the next question generated is more difficult and the probability of a correct answer being given is reduced by 0.1. Whenever a question is answered wrongly, the next question is of the same standard and the probability of a correct answer being given remains unchanged.

(i) Find the probability that the second question is answered correctly.

(1 mark)

(ii) Find the probability that the second question is answered correctly given that the third question is answered correctly. (2 marks)

(e) X is the number of heads when two coins are tossed. Find

(i) The expected number of heads. (1 mark)

(ii) $E(X^2)$ (1 mark)

(iii) $E(X^2 - X)$ (1 mark)

(f) A discrete random variable X has the given probability distribution.

x	1	2	3	4	5
P(X = x)	0.2	0.25	0.4	a	0.05

Find

(i) the value of a. (1 mark)

(ii) $P(1 \le X \le 3)$ (1 mark)

(iii) $P(X > 2)$ (1 mark)

(iv) $P(2 < X < 5)$ (1 mark)

(v) the mode (1 mark)

(g) X~ Geo(p) and it is known that $P(X = 2) = 0.21$ and $P < 0.5$. Find $P(X = 1)$. (3 marks)

(h) A random variable X has probability density function

$$f(x) = \begin{cases} 3x^k & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

(i) Find the value of k. (2 marks)

(ii) Find the mean of X. (2 marks)

(iii) Find the value of x such that $P(X \le x) = 0.5$. (2 marks)

QUESTION TWO (20 marks)

(a) A discrete random variable X has probability function

$$P(X=x)=\begin{cases}\dfrac{kx}{x^2+1} & x=2,\ 3\\[2ex] \dfrac{2kx}{x^2-1} & x=4,\ 5\\[2ex] 0 & otherwise\end{cases}$$

(i) Find k (2 marks)

(ii) Find the probability that X is less than 3 or greater than 4. (2 marks)

(iii) Find $F(3.2)$ (2 marks)

(iv) Find $Var(X)$ (6 marks)

(b) The probability that a student is awarded a distinction in a Statistics examination is 0.05. In a randomly selected group of 50 students what is the most likely number of students awarded a distinction? (5 marks)

(c) X follows a Poisson distribution with standard deviation 1.5. Find $P(X \geq 3)$ (3 marks)

QUESTION THREE (20 marks)

(a) A continuous random variable X has cumulative distribution function given by

$$F(x)=\begin{cases}0 & x<0\\ 2x-x^2 & 0\leq x\leq 1\\ 1 & x>1\end{cases}$$

(i) Find $P(X<\frac{1}{2})$ (2 marks)

(ii) Find the interquartile range of X. (6 marks)

(b) The length of worms in a garden are modeled by a uniform distribution between 1 cm and 5 cm.

(i) Find the standard deviation of this distribution. (2 marks)

(ii) Find the percentage of worms whose lengths lie within one standard deviation of the mean length. (3 marks)

(c) Machine A used for filling bags with ground coffee can be set to dispense any required mean weight of coffee per bag. At any setting the weight of coffee in a bag can be modeled by a normal distribution with a standard deviation of 1.95g.

(i) If the machine is set to dispense a mean weight of 128g of coffee per bag, calculate the percentage of bags that contain less than 125g. (2 marks)

To meet an official regulation the setting on a machine must be adjusted so that no more than 1% of bags contain less than 125g.

(ii) Calculate the smallest mean weight to which machine A should be set to meet the regulation. (3 marks)

(iii) Machine B will only just meet the regulation when it is set to dispense a mean weight of 128.5g. Assuming that the weight of coffee in a bag filled by machine B can be modeled by a normal distribution, calculate the standard deviation of this distribution. (2 marks)

QUESTION FOUR (20 marks)

(a) A probability density function of a random variable X is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0 & Otherwise \end{cases}$$

Compute

(i) The moment generating function (2 marks)

(ii) The mean (2 marks)

(iii) The variance (3 marks)

(b) A random variable X has the probability density function

$$f(x) = \begin{cases} \dfrac{2(x+1)}{(n+1)(n+2)}, & x = 0, 1, 2, \ldots, n \\ 0 & \textit{Otherwise} \end{cases}$$

(i) Find the conditional probability density function $f(x / X \leq m)$
Where $0 \leq m < n$ (4 marks)

(ii) Evaluate $P(X \leq r / X \leq m)$ where $0 \leq r \leq m$ (3 marks)

(c) It may be assumed that the breaking strength of cleaning towels is normally distributed with mean 50 units and standard deviation 8 units. Random samples of n towels are taken. The mean breaking strength for a sample is denoted by $\overline{X}$.

(i) State the distribution of $\overline{X}$ giving its mean and variance. (1 mark)

(ii) Find the probability that $\overline{X}$ exceeds 54 units in the case $n = 25$.(2 marks)

(iii) Find the smallest possible sample size if the probability that $\overline{X}$ exceeds 54 units is less than 0.01. (3 marks)

END OF EXAMINATION