KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE (BIOSTATISTICS)

SMA 573: STATISTICAL INFERENCE

DATE: Friday, 30th March, 2012 TIME: 2.00 p.m. – 5.00 p.m.

INSTRUCTIONS: Answer question **ONE** and any other **TWO** questions.

Question 1 (30 Marks)

(a) Define the following terms:

(i) A statistic (1 Mark)

(ii) Joint sufficient statistic (2 Marks)

(iii) Unbiased estimator (1 Mark)

(b) Let $x_1, x_2, x_3, \ldots, x_n$ be independently and identically distributed random variables with probability density function

$$f(x) = \begin{cases} P^x(1-P)^{1-x}, x = 0,1 \\ 0, elsewhere \end{cases}$$

Show that T=$\sum_{i=1}^{n} x_i$ is sufficient for P (7 Marks)

c) Suppose that the proportion θ of a defective item in a large manufactured lot is known and that the prior distribution of θ is a uniform distribution on the interval $[0,1]$. Let a random sample be taken from the lot. Determine the posterior distribution of θ given that the sampling for $x_i = \{1$, if the item is defective and 0 (zero), otherwise (8 marks)

(d) Let $x_1, x_2, x_3, ..., x_n$ be independently and identically distributed random variables form a normal population with mean μ and variance σ^2, where σ^2 is unknown. Construct a confidence interval for the mean μ (7 Marks)

(e) Suppose that there is a random sample $y_1, y_2, y_3, ..., y_n$ of size n from a population with mean θ, show that the sample mean $\bar{y}$ is unbiased for θ. (4 Marks)

Question 2 (20 Marks)

(a) Given an independent random sample of size n from a normal distribution with mean μ and variance σ^2, determine the maximum likelihood Estimators of μ and variance σ^2.

(7 Marks)

(b) Let $x_1, x_2, x_3, ..., x_n$ be independently and identically distributed random variables with probability density function

$$f(x) = \begin{cases} (\phi_2 - \phi_1)^{-1}, \phi_1 < X < \phi_2 \\ 0, elsewhere \end{cases}$$

Show that $Y_1 = \min(x_1, x_2, x_3, ..., x_n)$ and $Y_n = \text{Max}(x_1, x_2, x_3, ..., x_n)$ are jointly sufficient for ϕ_1 and ϕ_2 (7 Marks)

(c) Suppose $x_1, x_2, x_3, ..., x_n$ are independently and identically distributed random variables form a normal population with mean μ and variance σ^2, where σ^2 is unknown. Construct a confidence interval for the variance σ^2 (6 Marks)

Question 3 (20 Marks).

(a) Let $x_1, x_2, x_3, ..., x_n$ be a random sample from a normal population with mean θ and variance σ^2. Show that the statistic defined by

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

is an unbiased estimator of σ^2 where s^2 is the sample variance (6 Marks)

(b) Suppose $x_1, x_2, x_3, ..., x_n$ denote a random variables with probability density function

$$f(x) = \begin{cases} \dfrac{\theta^x e^{-\theta}}{x!}, x = 0,1,2,... \\ 0, elsewhere \end{cases}$$

Show that the family of the above distribution belongs to one parameter exponential family. (5 marks)

(c) Let $x_1, x_2, x_3, ..., x_m$ be a random sample of size m from a normal population with mean μ_1 and variance $\sigma^2{}_1$. Let $y_1, y_2, y_3, ..., y_n$ be a random sample from a normal population with mean μ_2 and variance $\sigma^2{}_2$. Assume that these samples are independent of each other. Construct a confidence interval for $\mu_1 - \mu_2$ (5 Marks)

(d) A random sample of size 100 trees of same age of a species A gave an average height of 50 mm with a standard deviation of 2.8mm while 150 trees of species B gave an average height of 55.5 mm and a standard deviation of 2.9mm. Construct a 95% confidence interval for the difference between the averages of the two tree species. (4 Marks)

Question 4 (20 Marks)

(a) Differentiate between a prior distribution and a posterior distribution (4 Marks)

(b) Let $x_1, x_2, x_3, ..., x_m$ be independently identically distributed random variables having common distribution $N(\theta_1, \theta_2{}^2)$ with both θ_1 and θ_2 unknown. Let the joint distribution of θ_1 and θ_2 be denoted by $h(\theta_1, \theta_2)$. Show that the statistic of $\bar{x}$ and Q=$\sum\left(x_i - \bar{x}\right)^2$ is Bayesian sufficient (16 Marks)