KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SCO 109: LINEAR ALGEBRA FOR COMPUTER SCIENCE

DATE: Monday 16th April 2012 **TIME:** 11.00a.m – 1.00p.m

Instructions: Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS) COMPULSORY

a) Find values of a and b given that $a\,(\frac{2}{13}x + \frac{-1}{3}z) + b(-2z) = 4x + 6z$ (2 marks)

b) Show if (0, 1, 0) a linear combination of (3, 0, -9) , (0, 5, 1) and (-1, 0, 0). (3 marks)

c) Given the points (1,-1, 4), (2, 1,-1) and (-1,-2,-3):

i. Find a general point on the plane through these points (2 marks)

ii. Find the equation of the plane. (3 marks)

d) Given that U= $(1, -3, \frac{1}{3})$ and V= (-1, -2, -3), evaluate $U \cdot V$ and $V \times U$ (3 marks)

e) Find the distance from origin to the plane $2x + 3\frac{1}{3}y - z = 2$. (2 marks)

f) Let $G = \{(x, y, z) | \; y + x + z \geq 0\}$ in R^3. Is G a subspace of R^3? Show. (4 marks)

g) Find how many of the vectors (1,0,-1,2), (2,1,1,0), (1,0,0,2) and (1,-2,4,3) are Linearly Independent by forming a matrix, reducing it to Echelon form and then drawing necessary conclusions. (4 marks)

h) Evaluate $\begin{vmatrix} -1 & 2 & \frac{1}{2} \\ \frac{2}{3} & -1 & 0 \\ 2 & -1 & 2 \end{vmatrix}$ (3 marks)

i) Let $F: R^n \rightarrow R^m$ be linear. Proof that Ker F is a subspace of R^n (4 marks)

QUESTION TWO

a) Define the rank and nullity of a linear transformation. [3 Marks]

A linear transformation f is defined as $f(x, y, z) = (x-y, x+y+z)$. Find

i. Kernel and nullity of f. [5 Marks]

ii. Image and Rank of f. [5 Marks]

b) Reduce the following matrix to the reduced row echelon form clearly showing the steps followed. Hence find the basis for the row space and the row rank of the matrix. [7 Marks]

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & -1 & 5 & 3 \\ 1 & 4 & -3 & -2 \\ 1 & -1 & 2 & 3 \end{bmatrix}$$

QUESTION THREE

a) Find the equation of the plane that passes through points (1, 1, 1), (2, 0, 2) and (-1,-1,-3). Hence find the distance of the plane from the origin. [7 Marks]

b) State the meaning of the phrase "*u is a linear combination of* $u_1, u_2, u_3, \ldots, u_n$". Hence determine if (5,3,1) is a linear combination of (5,1,0), (0,1,1) and (1,0,2). [7 Marks]

c) Show that the following sets are not vector spaces:

i. The set $V = \Re^2$ with the standard vector and scalar multiplication defined as

$c\,(u_1,u_2) = (u_1,cu_2)$ [3 Marks]

ii. The set V made up of 2×2 matrices whose elements are real numbers. [3 Marks]

QUESTION FOUR (20 MARKS)

a) Define a function $f: \Re^3 \rightarrow \Re^3$ by $f(x, y, z) = (x + y, y + z, x - z)$. Show that f is a linear transformation. Find the matrix of transformation of f. (9 Marks)

b) Let S be the set (1, 1, 1), (0, 1, 1), (0, 0, 1) of elements of $\Re^3$. Verify that S forms a basis for $\Re^3$ (6 Marks)

c) Solve the following system of linear equation (7 Marks)

$$2a - b + 4c - 2d = 3$$
$$a - 3c + 2d = 0$$
$$3a + 2b + c = 6$$
$$a + b + c - 2d = 1$$

QUESTION FIVE (20 MARKS)

a) Find a general point on the line (1, 1) and (-2, 1) (2 Marks)

b) Find the angle between the following planes $2y + 4x - 4z = 2$ and $2x - 4y - 4z = 4$ (4 Marks)

c) Find the equation of the plane parallel to the z axis and passing through (1, -2, 4) and (3, 2, -2) (5 Marks)

d) Given vectors $\breve{a} = (1, -2, 3)$, $\breve{b} = (2, 1, 5)$ and $\breve{c} = (3, 2, -1)$, find:

i. $\breve{a} \times \breve{b}$ (2 Marks)

ii. $\breve{a} \cdot (\breve{a} \times \breve{c})$ (3 Marks)

iii. $3\breve{b} \cdot (-5\breve{a})$ (2 Marks)

iv. $\breve{a} \times (2, -3)$ (2 Marks)