KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

SCO 111: DIFFERENTIAL CALCULUS FOR COMPUTER SCIENCE

DATE: TUESDAY, 17TH APRIL 2012 TIME: 11.00A.M – 1.00P.M

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO

QUESTION ONE (30 MARKS)

a) Evaluate the following limits

i. $\lim\limits_{x \to 0}\left(\frac{\sin 6x - 2x - 1}{2x}\right)$ (3 marks)

ii. $\lim\limits_{x \to \infty}\frac{3x+1}{\sqrt{x^2-1}}$ (3 marks)

iii. $\lim\limits_{x \to 2}\frac{x^2-4}{x-2}$ (3 marks)

b) Given that $x = 1 + t$, $y = 2t^2 - 4$ find $\frac{d^2y}{dx^2}$ (4 marks)

c) Differentiate the function $f(x) = \cos x$ using first principles (5 marks)

d) Differentiate the following functions with respect to x

i. $y = x^3 \tan^{-1}\left(\sqrt{x}\right)$ (4 marks)

ii. $y = x^{\sin x} + 10^x$ (4 marks)

e) A particle moves on the hyperbola $3x^2 - y^2 = 12$ in such a way that its y coordinate increases at a constant rate of 9 units per second. How first is its x coordinate changing when $x = 3$. (4 marks)

QUESTION TWO (20 MARKS)

a). If $y = e^{-2x}\cos 3x$,Show that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$ (5 marks)

b) i. State three conditions to be satisfied for a function to be continuous at find $x = a$ (3 marks)

ii. Determine whether the following function is continuous at $x = -1$

$$f(x) = \begin{cases} \frac{x^2 - 1}{x + 1} & \text{if } x < -1 \\ -2 & \text{if } x = -1 \\ -\sqrt{2 - x^2} & \text{if } x > -1 \end{cases}$$

(5 marks)

c). Distinguish the stationary points of the curve $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x - 1$ .Hence sketch the graph (7 marks)

QUESTION THREE (20 MARKS)

a). Find the equation of the tangent and normal to the curve Differentiate $x^3 - 3(xy^2) + y^4 = 1$ at the point (1,2) (7 marks)

b). If x and y are defined parametrically by $y = e^t \sin t$, $x = e^t \cos t$, show that $\frac{d^2y}{dx^2} = \frac{1}{e^t(\cos t - \sin t)^3}$ (7 marks)

c). If the radius of a sphere is increasing at the constant rate of 3 millimeters per second, how fast is the volume changing when the surface area is 10 square millimeters (6 marks)

QUESTION FOUR (20 MARKS)

a) Differentiate the following functions with respect to x

i. $y = \sqrt{2 + \sqrt{4 + \sqrt{3x}}}$ (4 marks)

ii. $y = \frac{\sinh^2 x}{\cosh^3 x}$ (4 marks)

iii. $y = \left(\sqrt{x^2 + 1}\right)\left(x^2 + 2x + 1\right)^2$ (3 marks)

iv. $y + \ln(xy) = 1$ (4 marks)

b) Differentiate $f(x) = \sqrt{x - 2}$ from first principles (5 marks)

QUESTION FIVE (20 MARKS)

a) Use $y = x^{1/4}$ to approximate $(81.6)^{1/4}$. (6 marks)

b) Evaluate the following limits

i. $\lim_{x \to -2} \left(\dfrac{x+2}{x^2 + x - 2} \right)$ (3 marks)

ii. $\lim_{x \to \infty} \left(\dfrac{3x^2 + 8x - 4}{2x^2 + 4x - 5} \right)$ (2 marks)

iii. $\lim_{x \to 2} \left(\dfrac{\sqrt{x^2 + 5} - 3}{x^2 - 2x} \right)$ (4 marks)

c) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if (5 marks)

$$y = \frac{\sin(2x)\cos x \tan^3 x}{\sqrt{1-3x}}$$