KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER SCIENCE

SMA 604: REPRESENTATION THEORY

DATE: Friday 2nd December 2011 TIME: 9.00a.m – 12.00p.m

INSTRUCTIONS

Answer question ONE and any other TWO questions

Question One

a) i) Define what meant by a representation of group G with representation space V. [2marks]

ii) Let a group G act on a set $A = \{a_1 a_2 ---, a_k\}$. choose for any field F, a k-dimension vector space V over F, and bais $\{v_1 v_2 ----, v_{16}\}$

If $x_{aj} = a_j$, define $t\ (x) v_j = v_j$ for every $x \in G.$ [3marks]

Show that T is a representation of G. [3marks]

b) i) Define the regular representation of a group G. [1mark]

ii) Write out the regular representation (as matrix) of $\mathbb{Z}_5$. [4marks]

c) i) If T is a representation define what is meant by the character χ of T. [1mark]

ii) Show that a character χ of G is irreducible if and only if $(\chi\ \chi,)$= 1. [4marks]

d) Let χ the character of G corresponding to the permutation representation of G. Show that $\chi_{(x)}$ counts the number of fixed points of x. [4marks]

e) Find the character table for $c_2\chi C_4$. [5marks]

f) The character table for the didral group of order 12, D_6 is

	$1C_1$	$2C_2$	$2C_3$	$1C_4$	$3C_5$	$3C_6$
χ_1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1
χ_3	1	-1	1	-1	1	-1
χ_4	1	-1	1	-1	-1	1
χ_5	2	1	-1	-2	0	0
χ_6	2	-1	-1	-2	0	0

Where χ_i denotes the i^{th} irreducible character and n_{cj} denotes the j^{th} conjugacy class containing n elements.

Use the table above to answer the following questions.

i) What is the order of the commutator subgroup of D_6? [1mark]

ii) What is the oarder of centre of D_6? [1mark]

iii) Express

	$1C_1$	$2C_2$	$2C_3$	$1C_4$	$3C_5$	$3C_6$
χ	8	-4	2	2	-2	0

as linear combination irreducible characters. [4marks]

<u>Question 2</u>

a) State without proof the Schur orthoganality relations. [2marks]

b) Let $\chi_1, \chi_2 \ldots\ldots \chi_k$ be the irreducible F- character of G.

Define what is meant by constituent and multiplicity of character χ of G.

[2marks]

c) i) Define the inner product (χ_1, χ_2) of two characters $\chi_{1\ and}\ \chi_{2.}$

[2marks]

ii) Show that if χ is a character of a_x group G and $\chi_1, \chi_2 \ldots\ldots \chi_k$ are irreducible characters of G, then $\chi = \sum_{j=1}^{k} (\chi, \chi_i)\ \chi_j$ [6marks]

d) Construct the character table for the cynic group of order 3. [6marks]

<u>Question 3</u>

a) i) Define the permutation representation of a group G. [2marks]

ii) Find a permutation representation of S_3 of degree 2. [4marks]

b) i) Define what is meant by a group acts doubly transitive on a set A.

[2marks]

ii) If G acts doubly transitive on a set A and θ is the character of the representation show that $\theta = l_a + \chi$, where χ is any irreducible character and $\chi\ \neq$ (the principal character) [3marks]

c) Show that if G acts o $A = \{a_1 a_2 - - - a_k\}$ with corresponding permutation representation T having characterθ, then $(\theta, 1_G)$ is the number of orbits of G and A. [6marks]

d) Show that irreducible F-characters are F-linearly independent. [3marks]

Question 4

a) Let G be a group and $T_1\ \ T_2 - - - T_k$ be all the irrecucible representations of G. Show that G is abelian if and only if every T_l is of degree 1. [5marks]

b) Let G be a non-abelian group of order 10.

i) What is the order of commentator subgroup G^1 of *G?* [2marks]

ii) How many irreducible characters does G posses and what are their degrees? [3marks]

c) Let $\chi_1, \chi_2 \ldots\ldots \chi_k$ be all the characters of a group *G*, and *e* be the character corresponding to the regular representation *R*.

i) Find (e, e) [2marks]

ii) Find (e, χ_i) [3marks]

d) i) Let T be a representation of G on a vector space V. Define what is meant by a subspace W and V is T-invariant (stable). [2marks]

iii) Let T be representation of G on V. Show that w of T (g) and image T(2) for every $g \epsilon G$ are T-invariant. [3marks]