KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

SMA 605: CODING THEORY

DATE: Thursday 5th April, 2012 **TIME:** 9.00 a.m. – 12.00 p.m.

INSTRUCTIONS

Answer QUESTION 1 and ANY OTHER TWO questions.

Question 1

(a) Consider the two codes shown below:

C_1 = {0000, 1100, 1010, 1001, 0110, 0101, 0011, 1111}

C2 = {000000, 111000, 001110, 110011}

(i) Determine the minimum distance of each code. [2 marks]

(ii) Determine the number or errors that each code can detect and correct. [2 marks]

(b) Consider the Hamming code defined by the parity-check matrix

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Decode the following received vectors

(i) x = 1110111 [2 marks]

(ii) y = 1111000 [2 marks]

(c) Find all code-words of the code determined by the parity-check matrix

$$H = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$ [4 marks]

(d) Prove the triangle inequality $d(x,y) \leq d(x,z) + d(z, y)$ where x, y, z are vectors in F^n.

[4 marks]

(e) Find a generator matrix of the code {0000, 1010, 0101, 1111} [4 marks]

(f) Write down the codewords of the cyclic code generated by $1 + x + x^2$ in $\mathbf{Z}_2^3[x]$, and find a parity check matrix for this code. [6 marks]

(g) Write down the irreducible factors of $x^5 - 1$ in $\mathbf{Z}_2[x]$, and hence determine all cyclic codes of length 5. [4 marks]

Question 2

(a) Show that a code C can correct all errors of weight up to t if and only if C has minimum distance $d(c) \geq 2t + 1$ [4 marks]

(b) (i) Write down all the codewords belonging to the linear code associated with the check matrix

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

[6 marks]

(ii) What are the parameters (n, k, d) for the code constructed in b(i)? [2 marks]

(c) Let C be the linear code defined by the parity-check matrix

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

If the word 110110 is received and only one error has been made, what is the intended codeword? [3 marks]

(d) (i) Explain what is meant by a cyclic code C. [2 marks]

(ii) Factorize $x^3 - 1$ into irreducible factors over $\mathbf{Z}_2$ and hence determine all the cyclic codes of length 3. [3 marks]

Question 3

(a) (i) Write down a parity check matrix for the Hamming code of length 15 by making the columns correspond to the binary representations of the integers 1, 2, . . ., 15 in order. [5 marks]

(ii) By using the matrix obtained in a (i) determine which of the following are codewords and correct those words which are not codewords, assuming that only one error has been made.

0 1 1 0 1 0 1 1 0 1 1 1 0 0 0

1 0 0 0 0 0 0 0 0 0 0 0 0 1 1

1 1 0 1 1 0 1 1 0 1 1 1 1 1 1 [5 marks]

(b) For a binary code whose generator matrix

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

(i) Find the generator matrix G in standard form for an equivalent code. [2 marks]

(ii) Find all codewords in this code. [5 marks]

(iii) Find the minimum distance of this code and the number of error it can detect and the number of errors it can correct. [3 marks]

Question 4

A binary code C is constructed as follows. To each block abc of three digits add three check digits xyz such that x = b + c, y = a + b, z = a + c

(a) Show that C is a linear code. [3 marks]

(b) List all the codes of C. [4 marks]

(c) Find the generator and check matrices of C in standard form. [6 marks]

(d) (i) Determine the minimum distance of C. [1 mark]

(ii) How many errors can C detect? [1 mark]

(iii) How many errors can C correct? [1 mark]

(e) How many cosets does C have? [2 marks]

(f) Find the syndromes of 100000 and 010001. [2 marks]

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