KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

INSTITUTION BASED PROGRAMME (IBP) AUGUST SESSION

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

SMA 637: NUMERICAL ANALYSIS IV

DATE: TUESDAY 3RD JANUARY 2012 TIME: 9.00A.M – 12.00P.M

INSTRUCTIONS: Answer any THREE Questions.

Question 1:

(a) Compute an approximation to $u(1), u'(1)$ and $u''(1.0)$ with the 'Taylor's series method of second order and step length h = 1.0 for the initial-value problem.

$u''' + 2u'' + u' = u + \cos t,\ \ 0 \le t \le 1$

$u(0) = 0, u'(0) = 1,\ \ u''(0) = 2$ after reducing it to the system of first order equation.

(10 marks)

(b) Use the classical Runge-kutta method of order four to solve the system of equations

$u' = -3u + 2v,\ \ u(0) = 0$
$v' = 3u - 4v,\ \ v'(0) = 0.5$

With h = 0.2 on the interval [0, 0.4]

(13 marks)

Question 2:

a) Determine a difference approximation of the problem $\frac{d}{dx}\left\{(1+x^2)\frac{dy}{dx}\right\} - y = x^2 + 1$

$y(-1) = y(1) = 0$

Find approximate value of *y(0)* using the step *h = 1* and *h = 0.5* and perform Richardson extrapolation. (12 marks)

(b) Determine the interval of absolute stability of the implicit 'Runge –Kutta method.

$$y_{n+1} = y_n + \tfrac{1}{4}(3k_1 + k_2)$$
$$k_1 = h\ f(x_n + \tfrac{h}{3},\ y_n + \tfrac{1}{3}k_1)$$
$$k_2 = h\ f(x_n + h,\ \ y_n + k_1)$$

When applied to the test equation $y' = \lambda y, \lambda < 0$ (11 marks)

Question 3

(a) The following initial-value problem is given $y' = 2s + 3y,\ \ y(0) = 1$

(i) If the error in $y(x)$ obtained from the first four terms of the Taylors series is to be less than $5\text{x}10^{-5}$ after rounding, find *x*. (6 marks)

(ii) Determine the number of terms in the Taylor series required to obtain results correct to $5\text{x}10^{-6}$ for $x \le 0.4$. (3 marks)

(iii) Use Taylor's series second order method to get $y(0.2)$ with step length *h = 0.1*. (6 marks)

(b) The multistep method $u_{j+1} - 3u_j + 2u_{j-1} = \tfrac{h}{2}(f_j - 3f_{j-1})$ is used to solve the test equation $u' = \lambda u, u(0) = 1$. Show that the solution of the difference equation does not approach the solution of the differential equation as $h \to 0$. (8 marks)

Question 4

(a) Discuss the shooting method for solving the boundary value problem

$y'' = f(x, y, y')x \in [9,6]$ with boundary condition $y(a) = \alpha, \quad y(b) = \beta$. (7 marks)

(b) Use the shooting method to solve the boundary value problem.

$$u'' = 2uu',\ \ 0 < x < 1$$
$$u(0) = 0.5,\ \ u(1) = 1$$

Use the two stage Runge-Kutta method

$$k_1 = \tfrac{h^2}{2} f(x_j, u_j, u'_j)$$

$$k_2 = \tfrac{h^2}{2} f(x_j + \tfrac{2}{3}h, u_j + \tfrac{2}{3} hu'_j + \tfrac{2}{3}k_1, u'_j + \tfrac{4}{3h}k_1)$$
$$u_{j+1} = u_j + hu'_j + \tfrac{1}{2}h(k_1 + 3k_2)$$

with $h = 0.25$, to solve the corresponding initial-value problem. Use Newton's method, assuming the starting value of the stope at $x = 0$, as $S^{(0)} = u'(0) = 0.3$. Perform two iterations and compare with the exact solution $u(x) = \dfrac{1}{(2-x)}$ (16 marks)

Question 5

(a) The Adam-Moulton Predictor-Corrector method is given by

$$P: y_{n+1} = y_n + \tfrac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

$$C: y_{n+1} = y_n + \tfrac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})$$

Use the P-C method, to find the solution of the initial-value problem

$y^1 = y - t^2$, $y(0) = 1$, at $t = 1.0$,

Taking h = 0.2. Use the following values as starting point,

$y(0.2) = 1.21859$, $y(0.4) = 1.46813$, $y(0.6) = 1.73779$.

Compare the result with the analytical solution. (10 marks)

(b) The formula

$$D_h = \tfrac{1}{2h}\left(3f(a) - 4f(a-h) + f(a-2h)\right)$$

is suitable to approximation $f'(a)$ where x is the last x-value in the table.

(i) State the truncation error $D_h - f'(a)$ as a power series in h. (2 marks)

(ii) Calculate $f'(2.0)$ as accurately as possible from the table.

x	$f(x)$
1.2	0.550630
1.3	0.577078
1.4	0.604826
1.5	0.634261
1.6	0.665766
1.7	0.699730
1.8	0.736559
1.9	0.776685
1.95	0.798129
2.0	0.820576

(8 marks)

(c) Show that Euler's method applied to $y' = \lambda y,\ y(0) = 1,\ \lambda < 0$ is stable for step-sizes $-2 < \lambda h < 0$ (stability means that $y_n \to 0$ as $n \to \infty$). (3 marks)

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