KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

INSTITUTIONAL BASED PROGRAMME (IBP) – AUGUST SESSION
EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE (STATISTICS)

SMA 666: TIME SERIES ANALYSIS

DATE: **Friday 30th December 2011** **TIME:** **9.00a.m -12.00p.m**

INSTRUCTIONS: Answer any three questions.

QUESTION ONE (23 marks)

a) Define an autoregressive process of a general order. [2marks]

b) Consider the process

$Y_t = \beta Y_{t-1} + Z_t \ , \ |\beta| < 1 \ and \ Z_t$

is white noise. Life successive substitution to express this process as the sum of infinite moving average process of random elements. Hence obtain its Autocorrelation function and sketch the corresponding correlogram. [21marks]

QUESTION TWO (23 marks)

a) Define the following terms

i) First order stationary process

ii) Second order stationary process [4marks]

b) Consider the process

$X_t = Z_t + \beta \sum_{i=1}^{\infty} Z_{t-1}$ where Z_t is a purely, random process. Show that X_t non stationary and find a filter that will make it stationary. [5marks]

c) Consider the process

$$Y_t = Y_{t-1} + \beta Y_{t-2} + Z_{t-2} + Z_{t-1} - 6Z_t$$

where β is a constant and $\{Z_t\}$ is a white noise process. Show that the process is invertible and find the value of β which will make the process stationary. [14marks

QUESTION THREE (23 marks)

a) State and explain the main stages in setting up the Box-Jenkins forecasting model. [4marks]

b) Let $X_1 \quad X_2 --- \quad X_{20}$, be the observed time series. Find minimum mean squared error forecast of x_{20+n} for $n \geq 1$. [10marks]

c) i) Define ARMA*(p,q)* process

iii) For ARMA (1,1) process (given by

$$Y_t = 0.5Y_{t-1} + Z_t + 0.16Z_{t-1}$$

obtain the forecast of 1, 2, 3 and 4 steps ahead using the difference equation form. Suggested by Box and Jenkins's. [9marks]

QUESTION FOUR (23 marks)

a) Let $f_x(\lambda)$ be the spectral density function of time seriesx_t with mean zero. Define a filter

$$Y_t = \sum_{j=-\infty}^{\infty} b_j X_{t-j}$$

Obtain an expression for the spectral density function of the time series Y_t [9marks]

b) Given an *AR (2)* process

$$X_t = \propto X_{t-1} + e_t$$

define a moving average filter

$$Y_t = \sum_{j=-\infty}^{\infty} b_j X_{t-j} \quad \text{, where}$$

$$b_j = \begin{matrix} \frac{1}{3} & , \ J = -2, -1, 0 \ 2 \\ 0 & else\ where \end{matrix}$$

obtain the spectral density function of Y_t [14marks]

QUESTION FIVE (23 marks)

a) Establish stationarity and inevitability of the process

$$X_t = X_{t-1} - \tfrac{3}{16}X_{t-2} + Z_t + \tfrac{1}{2}Z_{t-1} + \tfrac{1}{4}Z_{t-2}$$

where $\{Z_t\}$ is a white noise processes. [5marks]

b) For an AR (3) process given by

$$X_t = \frac{13}{20}X_{t-1} + \frac{1}{40}X_{t-2} - \frac{1}{20}X + Z_t$$

0.4 is one of the roots of the auxiliary equation of this process. Show that X_t is stationary and find its autocorrelation function. [15marks]

c) Find the autocorrelation function of the process.

$$X_t = Z_t - 0.5\, Z_{t-1} - 0.4\, Z_{t-2}$$

Where $\{Z_t\}$ is a purely random process. [3marks]