KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2010/2011
FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (COMPUTER TECHNOLOGY)

SCT 107: DISCRETE STRUCTURES I

DATE: FRIDAY 26TH NOVEMBER 2010 TIME: 11.00 A.M. - 1.00 P.M.

INSTRUCTIONS

Answer Question ONE and any other Two.

Question One (30 Marks)

a) Define the following:
   i) A proposition *(2marks)*
   ii) A tautology *(2marks)*

b) Let A and B be defined as follows:
   $A = \{5,7,11,13,...\}$
   $B = \{x \mid x \textit{ is a perfect square}\}$
   i). Describe A using set builder notation *(2marks)*
   ii). List the elements of B *(2marks)*
   iii). Compute $A \cap B$ *(1mark)*

c) Let $A = \{p,\{x,1\},n\}$
   $B = \{t,m\}$
   State **with reasons** whether each of the following is true or false.
   i). $x \in A$ $(2marks)$
   ii). $\{t\} \in B$ $(2marks)$
   iii). $\{t\} \subset B$ $(2marks)$

d) Suppose $A = \{x,\{1,y\}\phi\}$
   iv). What is the order of A? $(2marks)$
   v). Compute the power set of A. $(2marks)$

e) Suppose $f:\mathbb{R} \to \mathbb{R},\ f(x) = 2x^3 + 5$.
   i) Is f injective? *(2marks)*
   ii) Is f surjective? *(2marks)*
   iii) Construct two mappings g and h such that
   $g \circ h = f$ *(3marks)*

f) Write the following using quantification symbols:
   (i). The sum of any three consecutive integers is a multiple of three *(2marks)*
   (ii). The equation $3x^2 + 2x = 7$ has a real solution. $(2marks))$

Question Two (20 Marks)

a) i). Prove that for any 2 sets A and B

$$(A \cap B)^C = A^C \cup B^C$$ *(6marks)*

ii). What is the name given to this property? *(1mark)*

iii). State the dual property and illustrate the truth of the two statements using Venn diagrams. *(3marks)*

b) In a survey involving 61 students, it was found that 25 like hockey, 26 like soccer and 26 like basketball. 9 like both hockey and basketball, 11 like hockey and soccer while 7 like soccer and basketball. 9 like none of the three.

(i). Find the number of people who like at least one of the games. $(2marks)$

(ii). Find the number of people who like exactly one game. $(8marks)$

Question Three (20marks)

a) Show that a function f has an inverse if and only if f is a bijection. $(5marks)$

b) Suppose $f : A \to B$ and $g : B \to C$ are both surjective. Show that $g \circ f$ is also surjective. *(5marks)*

c) Explain what is meant by an even function and give an appropriate example. $(3marks)$

d) Let $f, g : \mathbb{R} \to \mathbb{R}$ such that $f(x) = x^3 + 2$ and $g(x) = 3x + 4$

i) Compute $g \circ f = h$ *(2Mark)*

ii) Get the inverses g^{-1}, f^{-1} and h^{-1} *(3 Marks)*

iii) Compute $f^{-1} \circ g^{-1}$,compare this to b(iii) above and make any relevant deduction. *(2 Marks)*

Question Four (20Marks)

a) Prove the following:

i). The product of two rational numbers is rational. $(4marks)$

ii). The sum of the first n squares (of natural numbers) equals $\frac{n(n+1)(2n+1)}{6}$ $(5marks))$

iii). $\sqrt{17}$ is irrational *(5marks)*

iv). The product of two irrational numbers is not necessarily irrational. **(Hint:** Give a counter example) $(4marks)$

b) Negate the following propositions:

i) The square of any real number is positive $(1mark)$

ii) There is a real number x such that $3x^2 + 5x = 0$ $(1mark)$

Question Five (20Marks)

a) Briefly explain what is meant by logical equivalence $(2marks)$

b) Let p be '*Jane reads the daily Nation*' ,q be '*Jane reads the Standard*' and r be '*Jane reads the people*'.

i) Write the following using connective symbols:
It is not true that either Jane reads the daily Nation or Jane doesn't read both the Standard and the People. *(3marks)*

ii) Write the following compound propositions in words:

- $r \wedge \neg(p \vee \neg q)$ $(2marks)$
- $(q \vee r) \leftrightarrow \neg q$ $(2marks)$

c) Consider the following statement:
If a matrix M is nonsingular, then M is invertible.

i). Rewrite the statement using another connective other than the implication. *(1mark)*

ii). Rewrite the statement using the phrase "**is a necessary condition**" *(2marks)*

iii). Rewrite the statement using the phrase "**is a sufficient condition**" *(2marks)*

iv). Write the inverse and contrapositive forms of this statement. $(6marks)$