KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER TECHNOLOGY

SCT 107: DISCRETE STRUCTURES I

DATE: FRIDAY 13TH APRIL 2012 **TIME: 2.00 P.M. – 4.00 P.M.**

INSTRUCTIONS

- Students from Computing and Information Technology to attempt questions in Section I.
- Students from Electronic and Electrical Engineering department to attempt questions in Section II

SECTION I *(COMPUTER SCIENCE STUDENTS)*

Attempt question One and any other Two Questions from the same questions

QUESTION ONE [COMPULSORY-30MKS]

(a) Use set builder method to specify the following sets

(i) The B set of even numbers equal to or greater than 4 but less than 10. (1mk)

(ii) The set D of all students who do Discrete structures I (1mk)

(b) Show that $(p \vee q) \wedge (\square\ p \vee r) \rightarrow (q \vee r)$ is a tautology. (4mks)

(c) Prove that for any two sets A and B, $(A \cup B)^c = A^c \cap B^c$. (4mks)

(d) Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - a_{n-2}$ for n=2, 3, 4...

and suppose that $a_0 = 3$ and $a_1 = 5$. What are a_2 and a_3? (2mks)

(e) State and prove the pigeonhole principle. (4mks)

(f) Let $A = \{x : -2 \le x \le 3, x \in \square\}$. List the members and give the cardinality of the following sets

(i) $B = \{x^2 : x \in A\}$ (2mks)

(ii) $C=\left\{\frac{x}{-2}:x\in A\right\}$ (2mks)

(g) Let $A=\{1,2,3,4\}$. With examples of relations on A, define

(i) A reflexive relation. (2mks)

(ii) A symmetric relation. (2mks)

(h) Prove that if $3n+2$ is odd then n is odd. (4mks)

(i) Let $A=\{x:5\le x<13, x \textit{ is prime}\}$. Find $\rho(A)$, the power set of A. (2mks)

QUESTION TWO [OPTIONAL-20MKS]

(a) Determine the truth values of the propositions

$p(-2,-3,4)$ and $p(4,3,-3)$ in the statement $xy\le z^2$ denoted by $p(x,y,z)$. (4mks)

(b) Express the following statements using predicates and quantifiers

(i) Every student of KU has done UCU 100. (2mks)

(ii) There is a student in Discrete Structures I class who cannot speak Arabic. (2mks)

(c) By translating into symbols and using rules of inference check the validity of the following argument.

It is not sunny this afternoon and it is colder than yesterday. If we will go swimming then it is sunny. If we do not go swimming then we will take a canoe trip. If we take a canoe trip, then we will be home by sunset. Hence we will be home by sunset. (5mks)

(d) Show that $p\leftrightarrow q$ and $(q\rightarrow p)\wedge(p\rightarrow q)$ are logically equivalent. (4mks)

(e) Write the inverse, converse and contrapositive of the statement

"If Jack is doing Discrete II, then he passed in Discrete I" (3mks)

QUESTION THREE (OPTIONAL-20 MKS)

(a) Use proof by cases to show that $x^3 + x$ is even $\forall\ x \in N$. (4mks)

(b) Prove that

(i) The sum of the first n squares of natural numbers is $\frac{n(n+1)(2n+1)}{6}$ (7mks)

(ii) $\sqrt{2}$ is irrational. (5mks)

(iii) The product of two rational numbers is rational. (4mks)

QUESTION FOUR [OPTIONAL-20MKS]

(a) Using the principle of inclusion-exclusion, find the number of integers between *1* and *200* inclusive which are divisible by *3*, *5* or 7. (9mks)

(b) Let $a_n = 2^n + 5(3^n)$ for $n = 0,1,2,\ldots.$ Show that $a_4 = 5a_3 - 6a_2$. (3mks)

(c) Let $g : A \to B$ and $f : B \to C$ be surjective functions. Show that $f_0 g$ is also surjective. (4mks)

(d) In a recent survey of 400 ICT students of a university college,100 were listed as studying Discrete Structures I and 150 studying Digital systems I. If 70 students were registered for both units, with the aid of a Venn diagram, find the number of students who were registered for Discrete structures I or Digital Systems I but not both. (4mks)

QUESTION FIVE (OPTIONAL-20MKS)

(a) Let $A = \{x \in \Re : x \neq 5\}$ and define f by $f(x) = \frac{2x}{x-5}$ $\forall x \in \Re$ .Show that f is 1-1 and determine a set B such that $f : A \to B$ is onto. (5mks)

(b) Let $f, g : \mathcal{R} \to \mathcal{R}$ such that $f(x) = x^2 + 1$ and $g(x) = \sqrt{3x}$.

(i) Compute $h = f_o g$. (2mks)

(ii) Find f^{-1}, g^{-1} and h^{-1}. (4mks)

(iii) Compute $g^{-1}{}_o f^{-1}$, compare this to (ii) and make any relevant deduction. (3mks)

(c) (i) Define a bijection. (2mks)

(ii) Let A be a set. Show that the identity function $1_A : A \to A$ where $1_A(x) = x \ \ \forall x \in A$ is a 1-1 correspondence. (4mks)

SECTION II (*SOFTWARE ENGINEERING)*

Attempt question One and any other Two questions from the same section

Question One(30 Marks)

a) Define the term power set. Let A={1,3,{1,3}}.
Evaluate the power set of A. (4 marks)

b) Let A {1, 2,3} and B={ 1 ,2}. Define a function f from A to B by f(1)2, f(2)=2 and f(3)=1. Is f surjective or injective? Proof (4 marks)

c) Proof by rnathematics induction that (6 marks)
∀n∈N, n^3 − n is divisible by 3.

d) Negate the following statement:
John is at the library or he is not at the book store. (2 marks)

e) Prove that a statement is logically equivalent to its contra positive. (4 Marks)

f) Draw the truth tables for P→(Q→R) and (P→Q)→R (4 marks)

g) Using a truth table illustrate: (6 marks)
i) A tautology
ii) A contradiction

Question 2 (20 Marks)

a) Define a mapping (2 marks)

b) Let A = {1,2,3,4,5} and B ={2,3,4,5,6}.

Let f :A B be defined by f (a) a+1 for all a A. Let g : B 4 C be defined by g(b) = 2b +3 Compute g_0f and g_0f 9 Marks

c) Let f:N→ N be defined by f(n) $3n+4$. Find f^1 (3marks)

d). Let f: XY be a bijection. Show that $f_0f^{1}=F^{1}{}_0 f=$ I the identity map. (6rnarks)

Question Three (20 Marks)

a) Evaluate A x B given that A={3,4} and B{3,6,4} (4Mar

b) For any three sets A,B,C, show that

A ∩ (B ∩ C) ≤ (A U B) ∩ (A U C) 6marks)

c) Out of 300 students taking discrete mathematics, 60 take coffee, 27 take cocoa, 36 take tea, 17 take tea only, 47 take chocolate only, 7 take chocolate and cocoa, 3 take chocolate, tea and cocoa, 20 take cocoa only, 2 take tea, coffee and chocolate 30 take coffee only, 9 take tea and chocolate whereas 12 take tea and coffee.

Find how many:

i. Take any beverage.

ii. Take cocoa and tea (10 marks)

Question Four (20 Marks)

a) Use mathematical induction to proof that $\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$ (10 marks)

b) Proof that the product of any two rational numbers is rational. (4 marks)

c) Use a contradiction to proof that the product of any non zero rational number and any irrational number is an irrational number. (6 marks)

Question 5 (20 marks)

a) Given the set A denoted by A = {2, 4, 17, 23}, state its cardinality (2 marks)

b) Use the induction principle to prove that every positive integer is either even or odd (6 marks)

c) Draw truth table for the following propositions(6 marks)
   i) $P \oplus Q$
   ii) $(P \land Q) \rightarrow Q$
   iii) $\neg P \land \neg Q$

d) Write an equivalence of the following statements: (6 marks)
   i) $\neg(P \lor Q)$ (Use the AND connective)
   ii) $\neg(P \land Q)$ (Use the OR connective)