KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR EDUCATION

<u>SST 102: DISCRETE MATHEMATICS</u>

DATE: Thursday, 29th March, 2012 TIME: 8.00 a.m. – 10.00 a.m.

<u>INSTRUCTIONS</u>:

Answer question **ONE** and any other **TWO** questions.

Question One (30 marks)

a) Illustrate the set $A \cap B$ on a Venn Diagram. (1 mark)

b) Prove that $A \cap B^c = A / B$ (4 marks)

c) Let A *and* B be disjoint finite sets, show that $n(A \cup B) = n(A) + n(B)$.

(3 marks)

d) i) Define a group (1 mark)

ii) Show that $(R, +)$ is an Abelian group (5 marks)

e) Use truth tables to show that $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology.

(4 marks)

f) Le $R(x, y, z)$ t denote the statement "$x + y = z$".

What are the truth values of the propositions $R(1,2,3)$ *and* $R(0,0,1)$?

(2 marks)

g) Fill in the following table on graph terminology

Type	Edges (Directed or Undirected	Multiple edges allowed?	Loops allowed?
Simple graph			
Multi graph			
Psendograph			
Simple Directed graph			
Directed Multi-graph			
Mixed graph			

(6 marks)

h) Use an adjacency matrix to represent the following graph

(1 mark)

i) Let R be a relation on a set A. There is a path of length n (a positive integer) from a to b iff $(a,b) \in R^n$. Prove. (3 marks)

Question Two (20 marks)

a) Prove that $(A \cap B)^C = A^C \cup B^C$ (4 marks)

b) In a survey of 60 people, it was found that 25 people read Newsweek magazine, 26 read Time, 26 read Fortune, 9 read both Newsweek and Fortune, 11 read both Time and Newsweek, 8 read both Time and Fortune, 3 read all magazines.

i) Find the number of people who read at least one of the three magazines. (3 marks)

ii) Fill in the correct number of people in a Venn diagram. (6 marks)

iii) Find the number of people who read exactly one magazine. (1 mark)

c) Define the following relation:

i) reflexive (1 mark)

ii) symmetric (1 mark)

iii) transitive (1 mark)

d) Show that the relation ‘ = ‘ on the set of real numbers is an equivalence relation.

(3 marks)

Question Three (20 marks)

a) Define the terms

i) Ring (2 marks)

ii) Division ring (1 mark)

iii) Field (1 mark)

b) Let $G = \{1, 2, 3, 4\}$. The binary operation on G is given by the following table

	1	2	3	4
1	1	2	3	4
2	2	4	2	4
3	3	2	1	4
4	4	4	4	4

i) Find the identity element, if any, in G.

ii) Which elements have inverses

iii) Are inverses unique?

iv) Is G associative?

(4 marks)

c) Show that the set $M(R)$ of 2 x 2 matrices over the real numbers is a ring with identity (9 marks)

d) Determine whether or not $\mathbb{Z}$, the set of integers, is a field. (3 marks)

Question Four (20 marks)

a) Which of the following are propositions?

i) Boston is the capital of Massachusetts

ii) Lagos is the capital of Nigeria

iii) Do not pass

iv) What exam is this?

(4 marks)

b) Le p *and* q be the propositions

p: The election is fair"

q: There will be no by-elections"

Express each of the following compound propositions as an English sentence

i) $\sim p \to \sim q$

ii) $p \leftrightarrow q$

iii) $\sim q \to \sim p$

iv) $\sim p \vee q$

(4 marks)

c) Show that $p \vee (q \wedge r)$ *and* $(p \vee q) \wedge (p \vee r)$ are logically equivalent

(6 marks)

d) Use truth tables to show that $(p \vee q) \wedge (\sim p \vee r) \to (q \vee r)$ is a tautology.

(6 marks)

Question Five (20 marks)

a) Define a graph (1 mark)

b) What are the degrees of the vertices in the graphs G and H displayed below?

(12 marks)

c) Illustrate the cycle $C_n, n \geq 3$ in a diagram. (4 marks)

d) Represent the following graph with an incidence matrix. (3 marks)