KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SST 103: LINEAR ALGEBRA I

DATE: Monday 2nd April, 2012 **TIME:** 11.00 a.m. – 1.00 p.m.

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (30 marks)

(a) Find the value of c such that $\begin{vmatrix} c & -3c \\ -1 & -c \end{vmatrix} = -2$ (4 marks)

(b) Given $A = \begin{pmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{pmatrix}$

(i) Reduce **A** to echelon form. (2 marks)

(ii) Reduce **A** to row – reduced echelon form. (3 marks)

(c) Let $A = \begin{pmatrix} 2 & 0 & 1 \\ 4 & 2 & -3 \\ 5 & 3 & 1 \end{pmatrix}$. Determine the inverse if it exists using the gaussian procedure. (6 marks)

(d) Solve using Cramer's rule.

$$\begin{aligned} 2x-5y+2z&=7 \\ x+2y-4z&=3 \\ 3x-4y-6z&=5 \end{aligned}$$

(5 marks)

(e) Let $\mathbf{x} = x_1\mathbf{i} + x_2\mathbf{j} + x_3\mathbf{k}$, $\mathbf{y} = y_1\mathbf{i} + y_2\mathbf{j} + y_3\mathbf{k}$, and $\mathbf{z} = z_1\mathbf{i} + z_2\mathbf{j} + z_3\mathbf{k}$ be vectors. Show that $\mathbf{x}.(\mathbf{y} \times \mathbf{z}) = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$ (4 marks)

(f) Let **X** be a subspace. $X = \{(x, y, w, z) : y + w + z = 0\}$. Find the dimension and the basis of **X**. (3 marks)

(g) Let $V = \Re^3$. Show that $\mathbf{U} = \{(x,\ y,\ 0) : x, y \in \Re\}$ is a subspace of V. (3 marks)

<u>QUESTION TWO</u> (20 marks)

(a) For which value of m will the vector $\mathbf{u} = (1, -2, m)$ in $\Re^3$ be a linear combination of $\mathbf{v} = (3, 0, -2)$ and $\mathbf{w} = (2, -1, -5)$? (5 marks)

(b) Find the angle between the vector $\mathbf{u} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ (4 marks)

(c) If $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$ and $\mathbf{B} = B_1\mathbf{i} + B_2\mathbf{j} + B_3\mathbf{k}$ prove that

(i) $\mathbf{A}.\mathbf{B} = A_1B_1 + A_2B_2 + A_3B_3$ (3 marks)

(ii) $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$ (2 marks)

(d) Determine whether or not the matrices

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad and \quad C = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

are linearly independent. (6 marks)

QUESTION THREE (20 marks)

(a) Solve using matrix inversion method.

$$
\begin{aligned}
x+y+2z&=1\\
x+2y-z&=-2\\
x+3y+z&=5
\end{aligned}
$$

(6 marks)

(b) Prove that for any two matrices **A** and **B**,

$(\mathbf{A}-\mathbf{B})(\mathbf{A}+\mathbf{B}) = \mathbf{A}^2 - \mathbf{B}^2$ if and only if $\mathbf{AB} = \mathbf{BA}$. (4 marks)

(c) Find the parametric equations of the line of intersection of the two planes

$x + y + z = 1$ and $2x - y + z = 1$ (4 marks)

(d) Find the equation of the plane through the points

$P_1 = (1, 0, 1)$, $P_2 = (2, 1, 3)$ and $P_3 = (-4, 1, -1)$ (6 marks)

QUESTION FOUR (20 marks)

(a) Given that $\mathbf{v} = (1, -2, 5)$, $\mathbf{e}_1 = (1, 1, 1)$, $\mathbf{e}_2 = (1, 2, 3)$ and $\mathbf{e}_3 = (2, -1, 1)$ check whether $\mathbf{v}$ is a linear combination of $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$. (6 marks)

(b) Let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \ldots \mathbf{u}_m \in \Re^n$. Let **A** be the set of all linear combinations of $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \ldots \mathbf{u}_m$. Prove that **A** is a subspace of $\Re^n$. (4 marks)

(c) Let **V** be a vector space over a Field **F** and suppose that **A** is a non – empty subset of **V**. State two necessary and sufficient conditions that **A** is a subspace of **V**. (2 marks)

(d) (i) State two conditions that a finite subset **B** of a vector space **V** must satisfy to be a basis for **V**. (2 marks)

(ii) Check whether $\mathbf{B} = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis for $\Re^3$ (6 marks)

QUESTION FIVE (20 marks)

(a) Determine whether or not the following vectors in $\Re^4$ are linearly dependent $\{(1, 3, -1, 4), (3, 8, -5, 7), (2, 9, 4, 23)\}$ (3 marks)

(b) Show that the xy – plane $W = \{(a, b, 0)\}, a, b \in \Re\}$ of $\Re^3$ is spanned by the vectors $\mathbf{u} = (2, -1, 0)$ and $\mathbf{v} = (1, 3, 0)$ (4 marks)

(c) Define $\Re^{n+}$ to be the set of n – tuples $(x_1, x_2, \ldots x_n)$ such that $x_i > 0$ for all i. Define addition and multiplication by

$$(x_1, x_2, \ldots x_n) = \oplus(y_1, y_2, \ldots y_n) = (x_1y_1, x_2y_2 \ldots x_ny_n)$$
$$\lambda \otimes (x_1, \ldots, x_n) = (x_1^\lambda, x_2^\lambda, \ldots x_n^\lambda)$$

Show that $\Re^{n+}$ is a vector space. (6 marks)

(d) Let **W** be the space generated by the polynomials

$$V_1 = t^3 - 2t^2 + 4t + 1$$
$$V_2 = 2t^3 - 3t^2 + 9t - 1$$
$$V_3 = t^3 + 6t - 5$$
$$V_4 = 2t^3 - 5t^2 + 7t + 5$$

Find a basis and the dimension of **W**. (7 marks)

END OF EXAMINATION