KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2017/2018

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ECONOMICS

EET 900: MICROECONOMICS I

DATE: Friday, 2nd February, 2018 TIME: 9.00 a.m. - 12.00 p.m.

INSTRUCTIONS: Answer **ALL** questions.

Question One

i. The total cost of a cost-minimizing firm is given as: $c(q, w_1, w_2) = a + (q + 0.5q^2)(w_1{}^{\beta} w_2^{\alpha} + (w_1^{\alpha} w_2^{\beta}) + w_2)$. a is a positive parameter, w_1 and w_2 are the unit prices of each of the two variable inputs and q is the output. Does the function meet the tenets of a cost-function? Demonstrate **(10 marks)**

ii. Consider a monopoly firm with demand function given as $y = 50 - p$ and cost function given as $c(y) = 5y + 0.5y^2$,

Determine the profit maximizing output and price levels **(3 marks)**

Suppose a price control is set on the firm requiring that its price be not higher than \$23, and the firm also faces a constraint that $y \leq 50 - p$. Analyze the profit maximization problem of the firm, giving all relevant cases. **(12 marks)**

Question Two

When solving optimization problems, one is likely to encounter challenges dealing with technology represented by a production function which is not differentiable. Using Kuhn Tucker Theorem, solve the following optimization problems.

i. Given a production function as $y = 3x_1 + 5x_2$, obtain the corresponding ordinary input demand functions and the cost function. **(15 marks)**

ii. Consider a firm with two plants each with cost functions $c_1(y_1) = 10y_1$ and $c_2(y_2) = y_2^2$ such that $y = y_1 + y_2$, and fixed factor prices. Derive the firm's cost function. **(10 marks)**

Question Three

(a) Let $X = R_+^2$. Verify that X satisfies all five properties required of a consumption set. **(2 marks)**

(b) A consumer has lexicographic preferences over $\mathbf{x} \in R_+^2$ if the relation $\succsim$ satisfies $\mathrm{x}^1 \succsim \mathrm{x}^2$ whenever $x_1^1 = x_1^2$, or $x_1^1 = x_1^2$ and $x_2^2 \geq x_2^2$.

(i) Sketch an indifference map for these preferences. **(2 marks)**

(ii) Can these preferences be represented by a continuous utility function? Why or why not? **(1 mark)**

(c) Suppose $u(\mathbf{x}) = \sum_{i=1}^{n} f_i(x_i)$ is strictly quasiconcave with $f_i'(xi)$ for all i. The consumer faces fixed prices $\mathbf{p} >> \mathbf{0}$ and has income $y > 0$. Assume $\mathbf{x}(\mathbf{p}, y) >> 0$.

(i) Show that if one good displays increasing marginal utility at x(p, y), all other goods must display diminishing marginal utility there. **(1 mark)**

(ii) Prove that if one good displays increasing marginal utility and all others diminishing marginal utility at x(p, y), then one good is normal and all other goods are inferior. **(1 mark)**

(iii) (iii) Show that if all goods display diminishing marginal utility at x(p, y), then all goods are normal. **(1 mark)**

(d) Consider a situation with two producers, A and B. Producer A has a production function $X = 0.4L^{0.25}$ where X is output and L the input of labour. Let Xp be the price of X, and W be the wage of labour. Producer B buys X and produces Y from it with a production function $Y = X^{0.5}$. The price of Y is $Y\,p$. The markets for Y and L are assumed to be competitive. Producer B acts as a competitor in the market for X. Required:

i) Derive the equilibrium values of L, X and Y. **(2 marks)**

ii) How do L, X and Y change as *W, Xp and Yp* change? **(2 marks)**

(e) Suppose initially a consumer has an income $m^0 = 100$ and faces the following prices of good x_1 and x_2: $p_1^0 = p_2^0 = 1$. If the government imposes a tax on the good x_1 so that p_1 *rises* to $p_1^1 = 4$. What amount of additional income would be needed to compensate the consumer so that he/she remains on the same level of utility as before the imposition of the tax? **(3 marks)**

Question Four

Ali, ''the crocodile'', lives in the dark water of Creek in the Arboretum just beside the certain campus. To the delight of the ducks there, Ali is a vegetarian. Ali earns income from being employed by the campus as a life guard at Creek. His task is to rescue students who fall into the water from time to time. He is especially busy around end of semester exams when more students than usual jump into water out of despair. When being rescued by Ali, usually their mood lights up quickly in anticipation of a cool selfie with a ''crocodile'' that can be shared on Facebook. By the time they relapse upon realizing that the water damaged their cell phone beyond repair, the authorities have arrived to help (with their cell phones). Anyway, let's not digress further. While we certainly appreciate Ali's work, our academic interest is focused on his consumption behaviour. Being a vegetarian, his main diet consists of '*'nduma*''. They have the inconvenient feature of getting stuck between his *72* teeth. So, he is also a rather heavy consumer of toothpicks.

Finally, there are his frequent gifts he purchases for Mathilda. Mathilda! She is the duck of the Arboretum who waddles most elegantly with her petite legs by delightfully moving well-shaped hips. He is really in love with her; at a platonic level of course so that nobody gets physically hurt.

We write $x_a, x_t, x_g \geq 0$ for the amounts of his consumption of ‘ ’*nduma*’’, toothpicks, and gifts, respectively. We assume for simplicity that these goods are infinitesimally divisible.

Let $x = (x_a, x_t, x_g))$. We assume that his utility function is given by $u(x_a, x_t, x_g) = x_a^{\alpha} x_t^{\theta} x_g^{1-\alpha-\theta}$ with $\alpha, \theta \in (0,1)$ and $\alpha + \theta < 1$. His income or wealth is denoted by w > 0. Finally, we denote by p_a; p_t; p_g > 0 the prices of ‘ ’*nduma*’’, toothpicks, and gifts, respectively, and p = (p_a; p_t; p_g).

(i) Use the Kuhn-Tucker approach to derive step-by-step the Walrasian demand function x(p;w). Verify also second-order conditions. **(2 marks)**

(ii) To be honest, we do not really know whether Ali has the Cobb-Douglas utility function stated above. Would Ali want to differently substitute a marginal amount of ‘ ’*nduma*’’ for some toothpicks when having a differentiable utility function different from the one above and optimally demanding positive amounts of all goods? Explain. **(1 mark)**

(iii) You would expect that the more ‘ ’*nduma*’’ Ali eats, the more they get stuck in his teeth and the more toothpicks he purchases. In light of such considerations, does it make sense to assume Ali has the utility function above? Explain. **(1 mark)**

(iv) Derive Ali's indirect utility function (denote it by v(p;w)). **(2 marks)**

(v) Verify that Ali satisfies Roy's identity with respect to ‘ ’*nduma*’’. **(1 mark)**

(vi) When Professor Yakuu interviews Ali about how exactly he arrives at his optimal consumption bundle, Ali expresses ignorance about maximizing utility subject to his budget constraint. Instead, he seems to minimize his expenditure on consumption such that he reaches a certain level of utility. A smart undergraduate student walks by and claims that this is clear evidence against the assumption of utility maximization in economics. Since Professor Yakuu hates Cobb-Douglas utility functions and boring calculations, he sends the student to you so that you can show him how expenditure minimization works. Again, use the Kuhn-Tucker approach to derive the Hicksian demand function. **(2 marks)**

(vii) Derive the expenditure function. Show that the expenditure function is homogeneous of degree 1 in prices, strictly increasing in $\bar{u}$ as well as nondecreasing and concave in the price of each good. **(2 marks)**

(viii) Professor Yakuu cannot compute the Hicksian demand using Kuhn-Tucker without a cup of tea. Unfortunately, Ali has no cup of tea to offer. Yet, he could offer Professor Yakuu his expenditure function. Is there a way to quickly calculate the Hicksian demand from the expenditure function without cup of tea? **(1 mark)**

(ix) Verify the (own price) Slutsky equation for the example of ‘ ’*nduma*’’. Which term in the (own price) Slutsky equation refers to the substitution effect? **(1 mark)**

(x) Because of the drought, the price of ‘ ’*nduma*’’ changes from. The campus is committed to keep Ali as well off as before the price change. The newly campus management turns to Professor Yakuu for advice on the exact amount to be deducted from the budget of the university and paid to Ali as compensation for the price change. Unfortunately, Professor Yakuu is so immersed in exciting new research that he is extremely slow in answering his email. Luckily, the Senior university administrator requests the university calculate the amount. **(2 marks)**