KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF ECONOMICS

EES 800: ECONOMETRICS

DATE: Friday, 12th May 2017 **TIME:** 9.00 a.m. - 12.00 p.m.

INSTRUCTIONS: Answer all the Four (4) questions. Each question carries 15 marks.

QUESTION ONE [15 Marks]

(a) A general linear model is given as follows: $Y = X\beta + e$; where Y is a Nx1 vector of the dependent variable; X is NxK matrix of observations on non- stochastic and linearly independent variables; β are unknown parameters and e is Nx1 vector of random error terms.

i. Assuming the first variable X_1 is a constant, derive the X matrix and its transpose X'.
ii. Derive the Ordinary Least Squares (OLS) estimators and its variance.
iii. Demonstrate the unbiased property of OLS estimators.

(b) A researcher obtained data from a sample of 392 cars. Four of the variables in the study include number of cylinders (X_1), engine displacement in Cubic inches (X_2), vehicle weight in Pounds (X3) and Miles per gallon (Y). Using OLS estimation method on the two models gave the following results;

MODEL ONE: $\hat{Y} = 42.9 - 3.558X_1$

(se)	(0.83)	(0.146)
{p- value}	0.000	0.000

MODEL TWO: $\hat{Y} = 44.4 - 0.268X_1 - 0.0127X_2 - 0.0057X_3$

(se)	(0.15)	(0.413)	(0.0083)	(0.00071)
{p- value}	(0.000)	(0.517)	(0.127)	(0.000).

i. Interpret the two models
ii. Using the given results and with reasons, highlight the OLS assumption that has been violated.
iii. Suggest a remedy for the problem identified in part (ii) above.

QUESTION TWO [15 marks]

a) Explain the problem of multicollinearity.

b) Given a model with four explanatory variables and a constant, illustrate using hypothetical results, the presence of near perfect multicollinearity in the model.

c) Enumerate the consequences of multicollinearity for the least squares estimator.

d) Part of data collected from a sample of 15 observations was estimated using OLS giving the following: $[X'X]^{-1} = \begin{bmatrix} 0.22191 & -0.0186 \\ -0.0186 & 0.0024 \end{bmatrix}$ and $X'Y = \begin{bmatrix} 186.4 \\ 1939.6 \end{bmatrix}$ while $e'e = 18.053$.

i. Use the information given above to find the OLS estimates for β_1 and β_2

ii. Determine the variance-covariance matrix of the OLS estimators

iii. Test the hypothesis that the Null $\beta_2 = 1$

QUESTION THREE [15 MARKS]

From economic theory, wage or earnings per hour (Y) can be expressed as a function of education in years (X_1), experience in years (X_2) and whether one resides in rural or urban area (X_3). Data was collected and residential area incorporated into the model as a dummy variable equal to one for urban worker and zero for rural worker. The OLS estimated equation is given as:

$$\hat{Y} = -9.914 + 1.234 X_1 + 0.133 X_2 + 1.524 X_3$$

$$(se) \quad (1.08) \quad (0.070) \quad (0.015) \quad (0.431)$$

a) Determine whether the estimates are significantly different from zero.

b) Construct a 95% confidence interval for education and experience variable.

c) Based on a 1000 sample of workers, establish the mean wage for rural workers and the mean wage for urban workers.

d) Suspecting the model suffers from Heteroscedasticity, the researcher utilized The Golgfeld-Quandt Test where the sample was separated into two distinct groups for rural and urban workers. Variance estimate for urban workers was found to be 31.824 while that of rural workers was 15.243. Given the upper critical values for a 5 % significance level F_{Uc} = 1.26, test whether the model suffers from Heteroscedasticity or not.

QUESTION FOUR [15 marks]

a. Consider the general model given as Y= Xβ + *e* with the variables Y and X as given in question one above. Use matrix algebra to demonstrate that the error term *e* has a constant variance

b. Assuming the model Y= Xβ + *e* suffers from heteroskedasticity

   i. Show that the Generalized Least Squares (GLS) estimators are unbiased.

   ii. Given that 3.27895 is a constant estimate of σ^2 for the model and that $[X'WX]^{-1} = \begin{bmatrix} 0.08484 & -0.0084 \\ -0.0084 & 0.00123 \end{bmatrix}$; $X'WY = \begin{bmatrix} 402.4 \\ 3476.2 \end{bmatrix}$ and

   $X'W^{-1}X = \begin{bmatrix} 9.75 & 95 \\ 95 & 1241.5 \end{bmatrix}$

   where $W = \Psi^{-1}$ and W = *diag* (1,1,1,1,1,1,1,1,4,4,4,4,4,4,4).

   Use the information to estimate β by GLS

   iii. Estimate the variance – covariance matrix of the GLS estimators

   iv. Test for the null hypothesis $\beta_2 = 1$ using GLS

TABLE F The *t*-Distribution

d.f.											
	0.900	0.700	0.500	0.300	0.200	0.100	0.050	0.020	0.010	α value	two tailed-test
	0.100	0.300	0.500	0.700	0.800	0.900	0.950	0.980	0.990	CL	
	0.450	0.350	0.250	0.150	0.100	0.050	0.025	0.010	0.005	α value	one tailed-test
	0.550	0.650	0.750	0.850	0.900	0.950	0.975	0.990	0.995	CL	
	Values of *t*										
1	0.158	0.510	1.000	1.963	3.078	6.314	12.706	31.821	63.657		
2	0.142	0.445	0.816	1.386	1.886	2.920	4.303	6.965	9.925		
3	0.137	0.424	0.765	1.250	1.638	2.353	3.182	4.541	5.841		
4	0.134	0.414	0.741	1.190	1.533	2.132	2.776	3.747	4.604		
5	0.132	0.408	0.727	1.156	1.476	2.015	2.571	3.365	4.032		
6	0.131	0.404	0.718	1.134	1.440	1.943	2.447	3.143	3.707		
7	0.130	0.402	0.711	1.119	1.415	1.895	2.365	2.998	3.499		
8	0.130	0.399	0.706	1.108	1.397	1.860	2.306	2.896	3.355		
9	0.129	0.398	0.703	1.100	1.383	1.833	2.262	2.821	3.250		
10	0.129	0.397	0.700	1.093	1.372	1.812	2.228	2.764	3.169		
11	0.129	0.396	0.697	1.088	1.363	1.796	2.201	2.718	3.106		
12	0.128	0.395	0.695	1.083	1.356	1.782	2.179	2.681	3.055		
13	0.128	0.394	0.694	1.079	1.350	1.771	2.160	2.650	3.012		
14	0.128	0.393	0.692	1.076	1.345	1.761	2.145	2.624	2.977		
15	0.128	0.393	0.691	1.074	1.341	1.753	2.131	2.602	2.947		
16	0.128	0.392	0.690	1.071	1.337	1.746	2.120	2.583	2.921		
17	0.128	0.392	0.689	1.069	1.333	1.740	2.110	2.567	2.898		
18	0.127	0.392	0.688	1.067	1.330	1.734	2.101	2.552	2.878		
19	0.127	0.391	0.688	1.066	1.328	1.729	2.093	2.539	2.861		
20	0.127	0.391	0.687	1.064	1.325	1.725	2.086	2.528	2.845		
21	0.127	0.391	0.686	1.063	1.323	1.721	2.080	2.518	2.831		
22	0.127	0.390	0.686	1.061	1.321	1.717	2.074	2.508	2.819		
23	0.127	0.390	0.685	1.060	1.319	1.714	2.069	2.500	2.807		
24	0.127	0.390	0.685	1.059	1.318	1.711	2.064	2.492	2.797		
25	0.127	0.390	0.684	1.058	1.316	1.708	2.060	2.485	2.787		
26	0.127	0.390	0.684	1.058	1.315	1.706	2.056	2.479	2.779		
27	0.127	0.389	0.684	1.057	1.314	1.703	2.052	2.473	2.771		
28	0.127	0.389	0.683	1.056	1.313	1.701	2.048	2.467	2.763		
29	0.127	0.389	0.683	1.055	1.311	1.699	2.045	2.462	2.756		
30	0.127	0.389	0.683	1.055	1.310	1.697	2.042	2.457	2.750		
40	0.126	0.388	0.681	1.050	1.303	1.684	2.021	2.423	2.704		
60	0.126	0.387	0.679	1.045	1.296	1.671	2.000	2.390	2.660		
120	0.126	0.386	0.677	1.041	1.289	1.658	1.980	2.358	2.617		
∞	0.126	0.385	0.674	1.036	1.282	1.645	1.960	2.326	2.576		