



KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2020/2021
SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF
MEDICAL LABORATORY SCIENCE

HSU 826: MEDICAL STATISTICS

DATE: FRIDAY 23RD JULY 2021

TIME: 9.00 A.M - 12.00 P.M

INSTRUCTIONS:

ANSWER ANY THREE (30 MARKS EACH)

Question One

- a) Pride et al compared the total cholesterol levels (mg/dl) between the patients with primary hypertension (PH) and normal patients (NT). She presents her data in the following table.

PH Patients	206	196	189	203	225	221	231	223	236
	208	265	203	199					
NT Patients	168	229	196	212	203	121	163	189	196
	142	179	212	239	218				

Test whether the two groups of patients have on average same cholesterol levels?

(7 marks)

- b) Ooko et al found that the number of serious gastrointestinal reactions reported to the Committee on Safety of Medicine was 479 for 68,500 prescriptions of an anti-inflammatory drug. This corresponds to a rate of 7 gastrointestinal reactions per 1000 prescriptions written. Using a Poisson model for probability, find the probability of
- Exactly 6 gastrointestinal reactions in 1000 prescriptions (2 marks)
 - No gastrointestinal reactions in 1000 prescriptions (3 marks)

- iii. Not more than four gastrointestinal reactions in 1000 prescriptions (3Marks)
- iv. At least three gastrointestinal reaction in 1000 prescriptions (2Marks)
- c) The analysis of variance for a randomized block design produced an ANOVA table entries shown below

Source	Df	ss	ms	F
Treatments	11	103		
Blocks			9.3	
Error				
Total	95	289.5		

- (i) Complete the ANOVA table (5 marks)
- (ii) Do the data provide sufficient evidence to indicate difference between blocks and between treatments? (2 marks)
- d) Susan used a portable devise among 550 patients to measure Bone Mineral Density (BMD) in the ankle in predicting a fracture. She used a Hologic estimated bone density of 0.57 as a cut off and presented the results in the table below

Blood group	Confirmed fracture		
	Present (D)	Not Present(D')	Total
BMD=.57(T)			52
BMD >.57(T')		55	
Total		153	

- I. Complete the table (2 marks)
- II. Determine the following parameters for this portable devise:-
- i. Test Sensitivity (2 marks)
- ii. Positive Predictive Value (2 marks)

Question Two

- a) Rael et al., studied 700 workers for ammonia as common breath metabolite and found a mean concentration in parts per billion (ppb) of 491 and standard deviation of 119.

Calculate the number of workers whose breath ammonia concentration was:

- (i) Between 291-610ppb (2 marks)
- (ii) Between 245-460ppb (3 marks)
- (iii) At least 500ppb (2 marks)
- (iv) Excessive ammonia in breath may be a sign of a metabolic disease. If 15 of these workers were referred for further examination as their levels were high, what was the cut off level above which the levels are critical? (4 marks)

- b) The ages of 50 blood donors in a given facility were presented as below

23	35	27	35	26	38	31	30	32	37	34	36
37	25	28	34	35	32	32	39	35	36	44	57
32	26	35	28	33	34	29	32	32	34	35	29
30	36	29	28	29	32	30	33	28	34	38	35
37	37										

Using above data

- I. Construct a frequency distribution using sturge's formula (4 marks)
- II. Using the frequency distribution in b (I) determine the
 - i. Mean (3 marks)
 - ii. Median (3 marks)
 - iii. Standard deviation (3 marks)
 - iv. Skewness(s_{kp}) (4 marks)
 - v. Would the distribution be said to be normally distributed? (2 marks)

Question Three

A random sample of the records of weights of single births was selected from each of the four populations. The results were presented as follows

Below 18 yrs	18-24	24-36	above 36
1.8	3.4	2.9	2.3
2.2	3.8	4.2	2.2
2.0	3.2	4.3	2.9
2.0	2.8	3.9	3.0
2.2	3.2	3.6	2.9
	3.6	3.4	2.7
		3.3	
		3.5	

- a) Provide sufficient evidence as to whether the four populations differ with respect to birth? (15 marks)
- b) If they do, perform appropriate statistical test to show the populations with difference. (15 marks)

Question Four

- a) Byron et al conducted a review of 12 subjects who were diagnosed with acute hepatitis A but were not hospitalized. Of interest was to use the age in years to predict bilirubin levels (mg/dL). The following were the data they collected:-

S.NO	Age (years)	Bilirubin (mg/dL)
1	72	13
2	58	5.8
3	58	5.6
4	45	2
5	44	7
6	52	5.8
7	81	13
8	42	1
9	57	16
10	50	5.6
11	59	8
12	78	3.8

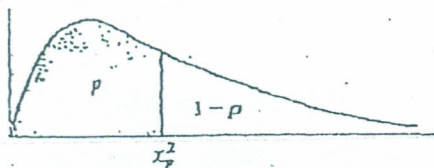
- Draw a scatter plot for the data (4 marks)
- Use the data to calculate the Spearman's Rank Correlation Coefficient (4 marks)
- Calculate the Covariance C_{xy} of the X and Y (3 marks)

- iv. Find the Karl Pearson's Correlation Coefficient r_{xy} (2 marks)
- v. Test whether the relationship between Age of the patient and bilirubin levels statistically significant? (3 marks)
- vi. Determine the equation of the regression line of least squares (3 marks)
- vii. Using the regression line in (vi) above, estimate the level of bilirubin in a patient of age 56 years. (1 mark)

b) An investigation was made on CD4 T cell count of administration of interleukin (IL-2) in addition to highly active antiretroviral therapy (HAART). The following table shows the CD4 T cell count at baseline and then again at 12 months of the (IL-2) therapy.

S.No	CD4 T cell count at entry	CD4 T cell count at end of follow up
1	170	250
2	63	160
3	105	308
4	189	362
5	301	462
6	350	240
7	162	229
8	193	427
9	172	423
10	285	141

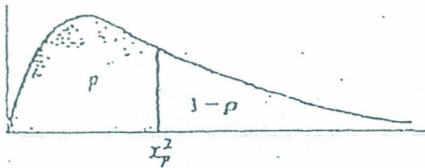
Perform an appropriate statistical test on the above data to compare the mean difference between the baseline data and the data after the follow up (10 marks)



Percentile Values χ^2_p for the Chi-Square Distribution with ν Degrees of Freedom

ν	$\chi^2_{.99}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.85}$	$\chi^2_{.80}$	$\chi^2_{.75}$	$\chi^2_{.70}$	$\chi^2_{.65}$	$\chi^2_{.60}$	$\chi^2_{.55}$	$\chi^2_{.50}$	$\chi^2_{.45}$	$\chi^2_{.40}$	$\chi^2_{.35}$	$\chi^2_{.30}$	$\chi^2_{.25}$	$\chi^2_{.20}$	$\chi^2_{.15}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$	$\chi^2_{.001}$																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																															
1	.0000	.0002	.0010	.0039	.0158	.102	.455	1.32	2.71	3.84	5.02	6.63	7.88	10.8	13.8	16.3	18.5	20.5	22.5	24.3	26.1	27.9	29.6	31.3	32.9	34.5	36.1	37.7	39.3	40.8	42.3	43.8	45.3	46.8	48.3	49.7	51.2	52.6	54.1	55.5	56.9	58.3	59.7	61.1	62.5	63.9	65.3	66.7	68.1	69.5	70.9	72.3	73.7	75.1	76.5	77.9	79.3	80.7	82.1	83.5	84.9	86.3	87.7	89.1	90.5	91.9	93.3	94.7	96.1	97.5	98.9	100.3	101.7	103.1	104.5	105.9	107.3	108.7	110.1	111.5	112.9	114.3	115.7	117.1	118.5	119.9	121.3	122.7	124.1	125.5	126.9	128.3	129.7	131.1	132.5	133.9	135.3	136.7	138.1	139.5	140.9	142.3	143.7	145.1	146.5	147.9	149.3	150.7	152.1	153.5	154.9	156.3	157.7	159.1	160.5	161.9	163.3	164.7	166.1	167.5	168.9	170.3	171.7	173.1	174.5	175.9	177.3	178.7	180.1	181.5	182.9	184.3	185.7	187.1	188.5	189.9	191.3	192.7	194.1	195.5	196.9	198.3	199.7	201.1	202.5	203.9	205.3	206.7	208.1	209.5	210.9	212.3	213.7	215.1	216.5	217.9	219.3	220.7	222.1	223.5	224.9	226.3	227.7	229.1	230.5	231.9	233.3	234.7	236.1	237.5	238.9	240.3	241.7	243.1	244.5	245.9	247.3	248.7	250.1	251.5	252.9	254.3	255.7	257.1	258.5	259.9	261.3	262.7	264.1	265.5	266.9	268.3	269.7	271.1	272.5	273.9	275.3	276.7	278.1	279.5	280.9	282.3	283.7	285.1	286.5	287.9	289.3	290.7	292.1	293.5	294.9	296.3	297.7	299.1	300.5	301.9	303.3	304.7	306.1	307.5	308.9	310.3	311.7	313.1	314.5	315.9	317.3	318.7	320.1	321.5	322.9	324.3	325.7	327.1	328.5	329.9	331.3	332.7	334.1	335.5	336.9	338.3	339.7	341.1	342.5	343.9	345.3	346.7	348.1	349.5	350.9	352.3	353.7	355.1	356.5	357.9	359.3	360.7	362.1	363.5	364.9	366.3	367.7	369.1	370.5	371.9	373.3	374.7	376.1	377.5	378.9	380.3	381.7	383.1	384.5	385.9	387.3	388.7	390.1	391.5	392.9	394.3	395.7	397.1	398.5	399.9	401.3	402.7	404.1	405.5	406.9	408.3	409.7	411.1	412.5	413.9	415.3	416.7	418.1	419.5	420.9	422.3	423.7	425.1	426.5	427.9	429.3	430.7	432.1	433.5	434.9	436.3	437.7	439.1	440.5	441.9	443.3	444.7	446.1	447.5	448.9	450.3	451.7	453.1	454.5	455.9	457.3	458.7	460.1	461.5	462.9	464.3	465.7	467.1	468.5	469.9	471.3	472.7	474.1	475.5	476.9	478.3	479.7	481.1	482.5	483.9	485.3	486.7	488.1	489.5	490.9	492.3	493.7	495.1	496.5	497.9	499.3	500.7	502.1	503.5	504.9	506.3	507.7	509.1	510.5	511.9	513.3	514.7	516.1	517.5	518.9	520.3	521.7	523.1	524.5	525.9	527.3	528.7	530.1	531.5	532.9	534.3	535.7	537.1	538.5	539.9	541.3	542.7	544.1	545.5	546.9	548.3	549.7	551.1	552.5	553.9	555.3	556.7	558.1	559.5	560.9	562.3	563.7	565.1	566.5	567.9	569.3	570.7	572.1	573.5	574.9	576.3	577.7	579.1	580.5	581.9	583.3	584.7	586.1	587.5	588.9	590.3	591.7	593.1	594.5	595.9	597.3	598.7	600.1	601.5	602.9	604.3	605.7	607.1	608.5	609.9	611.3	612.7	614.1	615.5	616.9	618.3	619.7	621.1	622.5	623.9	625.3	626.7	628.1	629.5	630.9	632.3	633.7	635.1	636.5	637.9	639.3	640.7	642.1	643.5	644.9	646.3	647.7	649.1	650.5	651.9	653.3	654.7	656.1	657.5	658.9	660.3	661.7	663.1	664.5	665.9	667.3	668.7	670.1	671.5	672.9	674.3	675.7	677.1	678.5	679.9	681.3	682.7	684.1	685.5	686.9	688.3	689.7	691.1	692.5	693.9	695.3	696.7	698.1	699.5	700.9	702.3	703.7	705.1	706.5	707.9	709.3	710.7	712.1	713.5	714.9	716.3	717.7	719.1	720.5	721.9	723.3	724.7	726.1	727.5	728.9	730.3	731.7	733.1	734.5	735.9	737.3	738.7	740.1	741.5	742.9	744.3	745.7	747.1	748.5	749.9	751.3	752.7	754.1	755.5	756.9	758.3	759.7	761.1	762.5	763.9	765.3	766.7	768.1	769.5	770.9	772.3	773.7	775.1	776.5	777.9	779.3	780.7	782.1	783.5	784.9	786.3	787.7	789.1	790.5	791.9	793.3	794.7	796.1	797.5	798.9	800.3	801.7	803.1	804.5	805.9	807.3	808.7	810.1	811.5	812.9	814.3	815.7	817.1	818.5	819.9	821.3	822.7	824.1	825.5	826.9	828.3	829.7	831.1	832.5	833.9	835.3	836.7	838.1	839.5	840.9	842.3	843.7	845.1	846.5	847.9	849.3	850.7	852.1	853.5	854.9	856.3	857.7	859.1	860.5	861.9	863.3	864.7	866.1	867.5	868.9	870.3	871.7	873.1	874.5	875.9	877.3	878.7	880.1	881.5	882.9	884.3	885.7	887.1	888.5	889.9	891.3	892.7	894.1	895.5	896.9	898.3	899.7	901.1	902.5	903.9	905.3	906.7	908.1	909.5	910.9	912.3	913.7	915.1	916.5	917.9	919.3	920.7	922.1	923.5	924.9	926.3	927.7	929.1	930.5	931.9	933.3	934.7	936.1	937.5	938.9	940.3	941.7	943.1	944.5	945.9	947.3	948.7	950.1	951.5	952.9	954.3	955.7	957.1	958.5	959.9	961.3	962.7	964.1	965.5	966.9	968.3	969.7	971.1	972.5	973.9	975.3	976.7	978.1	979.5	980.9	982.3	983.7	985.1	986.5	987.9	989.3	990.7	992.1	993.5	994.9	996.3	997.7	999.1	1000.5	1001.9	1003.3	1004.7	1006.1	1007.5	1008.9	1010.3	1011.7	1013.1	1014.5	1015.9	1017.3	1018.7	1020.1	1021.5	1022.9	1024.3	1025.7	1027.1	1028.5	1029.9	1031.3	1032.7	1034.1	1035.5	1036.9	1038.3	1039.7	1041.1	1042.5	1043.9	1045.3	1046.7	1048.1	1049.5	1050.9	1052.3	1053.7	1055.1	1056.5	1057.9	1059.3	1060.7	1062.1	1063.5	1064.9	1066.3	1067.7	1069.1	1070.5	1071.9	1073.3	1074.7	1076.1	1077.5	1078.9	1080.3	1081.7	1083.1	1084.5	1085.9	1087.3	1088.7	1090.1	1091.5	1092.9	1094.3	1095.7	1097.1	1098.5	1099.9	1101.3	1102.7	1104.1	1105.5	1106.9	1108.3	1109.7	1111.1	1112.5	1113.9	1115.3	1116.7	1118.1	1119.5	1120.9	1122.3	1123.7	1125.1	1126.5	1127.9	1129.3	1130.7	1132.1	1133.5	1134.9	1136.3	1137.7	1139.1	1140.5	1141.9	1143.3	1144.7	1146.1	1147.5	1148.9	1150.3	1151.7	1153.1	1154.5	1155.9	1157.3	1158.7	1160.1	1161.5	1162.9	1164.3	1165.7	1167.1	1168.5	1169.9	1171.3	1172.7	1174.1	1175.5	1176.9	1178.3	1179.7	1181.1	1182.5	1183.9	1185.3	1186.7	1188.1	1189.5	1190.9	1192.3	1193.7	1195.1	1196.5	1197.9	1199.3	1200.7	1202.1	1203.5	1204.9	1206.3	1207.7	1209.1	1210.5	1211.9	1213.3	1214.7	1216.1	1217.5	1218.9	1220.3	1221.7	1223.1	1224.5	1225.9	1227.3	1228.7	1230.1	1231.5	1232.9	1234.3	1235.7	1237.1	1238.5	1239.9	1241.3	1242.7	1244.1	1245.5	1246.9	1248.3	1249.7	1251.1	1252.5	1253.9	1255.3	1256.7	1258.1	1259.5	1260.9	1262.3	1263.7	1265.1	1266.5	1267.9	1269.3	1270.7	1272.1	1273.5	1274.9	1276.3	1277.7	1279.1	1280.5	1281.9	1283.3	1284.7	1286.1	1287.5	1288.9	1290.3	1291.7	1293.1	1294.5	1295.9	1297.3	1298.7	1299.9	1301.3	1302.7	1304.1	1305.5	1306.9	1308.3	1309.7	1311.1	1312.5	1313.9	1315.3	1316.7	1318.1	1319.5	1320.9	1322.3	1323.7	1325.1	1326.5	1327.9	1329.3	1330.7	1332.1	1333.5	1334.9	1336.3	1337.7	1339.1	1340.5	1341.9	1343.3	1344.7	1346.1	1347.5	1348.9	1350.3	1351.7	1353.1	1354.5	1355.9	1357.3	1358.7	1360.1	1361.5	1362.9	1364.3	1365.7	1367.1	1368.5	1369.9	1371.3	1372.7	1374.1	1375.5	1376.9	1378.3	1379.7	1381.1	1382.5	1383.9	1385.3	1386.7	1388.1	1389.5	1390.9	1392.3	1393.7	1395.1	1396.5	1397.9	1399.3	1400.7	1402.1	1403.5	1404.9	1406.3	1407.7	1409.1	1410.5	1411.9	1413.3	1414.7	1416.1	1417.5	1418.9	1420.3	1421.7	1423.1	1424.5	1425.9	1427.3	1428.7	1430.1	1431.5	1432.9	1434.3	1435.7	1437.1	1438.5	1439.9	1441.3	1442.7	1444.1	1445.5	1446.9	1448.3	1449.7	1451.1	1452.5	1453.9	1455.3	1456.7	1458.1	1459.5	1460.9	1462.3	1463.7	1465.1	1466.5	1467.9	1469.3	1470.7	1472.1	1473.5	1474.9	1476.3	1477.7	1479.1	1480.5	1481.9	1483.3	1484.7	1486.1	1487.5	1488.9	1490.3	1491.7	1493.1	1494.5	1495.9	1497.3	1498.7	1499.9	1501.3	1502.7	1504.1	1505.5	1506.9	1508.3	1509.7	1511.1	1512.5	1513.9	1515.3	1516.7	1518.1	1519.5	1520.9	1522.3	1523.7	1525.1	1526.5	1527.9	1529.3	1530.7	1532.1	1533.5	1534.9	1536.3	1537.7	1539.1	1540.5	1541.9	1543.3	1544.7	1546.1	1547.5	1548.9	1550.3	1551.7	1553.1	1554.5	1555.9	1557.3	1558.7	1560.1	1561.5	1562.9	1564.3	1565.7	1567.1	1568.5	1569.9	1571.3	1572.7	1574.1	1575.5	1576.9	1578.3	1579.7	1581.1	1582.5	1583.9	1585.3	1586.7	1588.1	1589.5	1590.9	1592.3	1593.7	1595.1	1596.5	1597.9	1

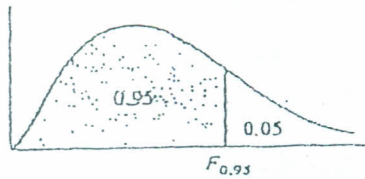
Percentile Values χ_p^2 for the Chi-Square Distribution with ν Degrees of Freedom



ν	$\chi_{.99}^2$	$\chi_{.95}^2$	$\chi_{.90}^2$	$\chi_{.85}^2$	$\chi_{.80}^2$	$\chi_{.75}^2$	$\chi_{.70}^2$	$\chi_{.65}^2$	$\chi_{.60}^2$	$\chi_{.55}^2$	$\chi_{.50}^2$	$\chi_{.45}^2$	$\chi_{.40}^2$	$\chi_{.35}^2$
1	.0000	.0002	.0010	.0039	.0158	.102	.455	1.32	2.71	3.84	5.02	6.63	7.88	10.8
2	.0100	.0201	.0506	.103	.211	.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6	13.8
3	.0717	.115	.216	.352	.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8	16.3
4	.207	.297	.484	.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9	18.5
5	.412	.554	.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7	20.5
6	.616	.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5	22.5
7	.929	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3	24.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0	26.1
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6	27.9
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2	29.6
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8	31.3
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3	32.9
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8	34.5
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3	36.1
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8	37.7
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3	39.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7	40.8
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2	42.3
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6	43.8
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0	45.3
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4	46.8
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8	48.3
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2	49.7
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6	51.2
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9	52.6
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3	54.1
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6	55.5
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0	56.9
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3	58.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7	59.7
40	20.7	22.2	24.4	26.5	29.1	33.7	39.3	45.6	51.8	55.8	59.3	63.7	66.8	73.4
50	28.0	29.7	32.4	34.8	37.7	42.9	49.3	56.3	63.2	67.5	71.4	76.2	79.5	86.7
60	35.5	37.5	40.5	43.2	46.5	52.3	59.3	67.0	74.4	79.1	83.3	88.4	92.0	99.6
70	43.3	45.4	48.8	51.7	55.3	61.7	69.3	77.6	85.5	90.5	95.0	100	104	112
80	51.2	53.5	57.2	60.4	64.3	71.1	79.3	88.1	96.6	102	107	112	116	125
90	59.2	61.8	65.6	69.1	73.3	80.6	89.3	98.6	108	113	118	124	128	137
100	67.3	70.1	74.2	77.9	82.4	90.1	99.3	109	118	124	130	136	140	149

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 1 (1966), Table B, pages 137 and 138, by permission.

95th Percentile Values (0.05 Levels), $F_{0.95}$ for the F Distribution



ν_1 degrees of freedom in numerator
 ν_2 degrees of freedom in denominator

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.10	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table F, p. 172.

99th Percentile Values (0.01 Level) $F_{0.99}$ for the F Distribution



ν_1 degrees of freedom in numerator
 ν_2 degrees of freedom in denominator

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120
1	4052	5000	5403	5625	5764	5859	5928	5981	6023	6056	6106	6157	6209	6235	6261	6287	6313	6339
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.82	2.66	2.58	2.50	2.42	2.33	2.23
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table J, page 180, by permission.

FORMULA FOR USE IN BIOSTATISTICS AND BIOMETRICS

$$\text{Mean } \bar{x} = \frac{\sum fix_i}{\sum f_i} = \text{or } A + \left(\frac{\sum f_i u_i}{\sum f_i} \right) k;$$

$$\text{Median } \tilde{x} = L_i + \frac{h}{f} \left[\frac{N}{2} - C \right];$$

$$\text{Mode } M_o = L_i + \left[\frac{f_m - f_1}{2f_m - f_1 - f_2} \right] h_i$$

$$D = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}} = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2};$$

$$\text{Coefficient of Variation } CV = \frac{s}{\bar{x}} = \frac{\sigma}{\bar{x}}.$$

$$\text{Standard error; } SE = \frac{s}{\sqrt{n-1}}, n < 30;$$

$$SE = \frac{s}{\sqrt{n}}, n \geq 30$$

$$\text{Quartile Coefficient of Variation, } V_Q = \frac{Q_3 - Q_1}{Q_3 + Q_1};$$

$$\text{Binomial Distribution; } P(x) = {}^n C_x p^x q^{n-x}$$

$$\text{Poisson distribution} = P(x) = \frac{e^{-m} m^x}{x!};$$

$$\text{Standard Normal Variable } Z = \frac{x - \mu}{\sigma}$$

$$\text{Confidence Interval, } CI = \bar{x} \pm \pm Z_{\frac{\alpha}{2}}$$

$$CI = \bar{x} \pm \pm T_{\frac{\alpha}{2}}$$

$$\text{Test of Significance, } Z = (\bar{x} - \mu) / (\sigma / \sqrt{n});$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\left(2 \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)}$$

$$\frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} \text{ where } \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} \right)}$$

$$\text{Test of Significance, } T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}};$$

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sigma \sqrt{\left[\frac{1}{n_1} + \frac{1}{n_2} \right]}} \text{ where } \sigma = \sqrt{\left[\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} \right]}$$

$$\text{Conditional Probability } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\text{Bayes' theorem } P(A_k/A) = \frac{P[A_k \cap A]}{P(A)} = \frac{P(A_k)P(A/A_k)}{\sum_{j=1}^n P(A_j)P(A/A_j)}$$

$$\text{Coefficient of skewness } S_{k_p} = \frac{3(\bar{x} - \tilde{x})}{s}; \text{ or } S_k = \frac{(x - \bar{x})^3}{NS^3} f_i$$

$$\text{Kurtosis } Kur = \frac{\sum f_i (x_i - \bar{x})^4}{NS^4}$$

$$s = 1 + 3.322 \log_{10} n$$

$$t = \frac{r}{k}$$

FORMULAE FOR USE IN BIOSTATISTICS AND BIOMETRICS II (HEH 301) EXAM

1. Covariance $C_{xy} = \frac{\sum (y - \bar{y})(x - \bar{x})}{N} = \frac{N \sum xy - \sum x \sum y}{N^2}$

2. Correlation $r_{xy} = \frac{C_{xy}}{s_x s_y} = \frac{\sum (x - \bar{x})(y - \bar{y})}{N s_x s_y} = \frac{N \sum xy - \sum x \sum y}{N^2 s_x s_y} = \frac{N \sum xy - \sum x \sum y}{\sqrt{(N \sum x^2 - (\sum x)^2)(N \sum y^2 - (\sum y)^2)}}$

3. Regression line of least squares

$y = a + bx$; $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} = \frac{N \sum xy - \sum x \sum y}{(N \sum x^2 - (\sum x)^2)} = r_{xy} \cdot \frac{s_y}{s_x}$ $a = \bar{y} - b\bar{x}$

4. Chi-square

$\chi^2 = \sum \left(\frac{(O_i - E_i)^2}{E_i} \right) = \frac{n(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)}$

5. Relative risk $\frac{a/(a+b)}{c/(c+d)} = \frac{a(c+d)}{c(a+b)}$

$S_c = 1 - \frac{6 \sum D^2}{n(n^2 - 1)}$

6. Odds ratio $\frac{ad}{bc}$

7. Mantel Haenzel

$\chi_{MH}^2 = \frac{(\sum_{i=1}^k a_i - \sum_{i=1}^k e_i)^2}{\sum_{i=1}^k v_i}$ $e_i = \frac{(a_i + b_i)(a_i + c_i)}{n_i}$

$v_i = \frac{(a_i + b_i)(c_i + d_i)(a_i + c_i)(b_i + d_i)}{n_i^2 (n_i - 1)}$

$t = \sqrt{\frac{n-2}{1-r^2}}$

8. Anova

$CM = \frac{(\sum y_i)^2}{n}$; $SS(Total) = \sum y_i^2 - CM$

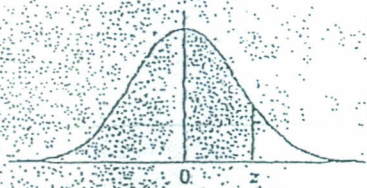
$t_{HSD} = q_{\alpha, K, N-K} \sqrt{\frac{MSE}{2} \left(\frac{1}{n_a} + \frac{1}{n_b} \right)}$

$SST = \frac{T_1^2}{n_1} + \frac{T_2^2}{n_2} + \dots + \frac{T_p^2}{n_p} - CM$

$SSE = SS(Total) - SSTreatment - SSBlock$ or $SS(Total) - SST - SSB$

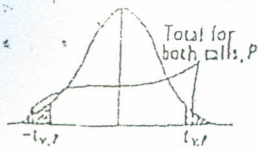
$MST = \frac{SST}{p-1}$ $MSE = \frac{SSE}{n-p}$; $F = \frac{MST}{MSE}$

$MSB = \frac{SSB}{b-1}$ $MSE = \frac{SSE}{n-p-b+1}$



Areas under the Standard Normal Curve from 0 to z

z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Table A.1 Percentage points of the t distribution

The function tabulated is $t_{\alpha/2}$, the value exceeded in both directions with probability P in a t distribution with ν degrees of freedom (the 100P percentage point).

Degrees of freedom, ν	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05	0.02	0.01	0.001
1	0.158	0.325	0.510	0.727	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	0.147	0.289	0.445	0.617	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	0.137	0.277	0.424	0.584	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.134	0.271	0.414	0.569	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.132	0.267	0.408	0.559	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.131	0.265	0.404	0.553	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.130	0.263	0.402	0.549	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.130	0.262	0.399	0.546	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.129	0.261	0.398	0.543	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.129	0.260	0.397	0.542	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.129	0.260	0.396	0.540	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.457
12	0.128	0.259	0.395	0.539	0.695	0.873	1.085	1.356	1.782	2.179	2.681	3.055	4.318
13	0.128	0.259	0.394	0.538	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.128	0.258	0.393	0.537	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.128	0.258	0.393	0.536	0.691	0.866	1.071	1.341	1.753	2.131	2.602	2.947	4.073

APPENDIX TABLES

0.128	0.258	0.392	0.535	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
0.128	0.257	0.392	0.534	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
0.127	0.257	0.392	0.534	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
0.127	0.257	0.391	0.533	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
0.127	0.257	0.391	0.533	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
0.127	0.257	0.391	0.532	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
0.127	0.256	0.390	0.532	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
0.127	0.256	0.390	0.532	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
0.127	0.256	0.390	0.531	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
0.127	0.256	0.389	0.531	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
0.127	0.256	0.389	0.530	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
0.126	0.255	0.388	0.529	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
0.126	0.254	0.387	0.527	0.679	0.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
0.126	0.254	0.386	0.526	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
0.126	0.253	0.385	0.524	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

A.1 DISTRIBUTION

† from Table III of Fisher and Yates (1963) by permission of the authors and publishers.

TABLE II Percentage Points of the Studentized Range for 2 Through 20 Treatments Upper 5% Points

Error df	2	3	4	5	6	7	8	9	10
1	17.97	26.98	32.82	37.08	40.41	43.12	45.40	47.36	49.07
2	6.08	8.33	9.80	10.88	11.74	12.44	13.03	13.54	13.99
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15
17	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65
120	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56
∞	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47

Error df	11	12	13	14	15	16	17	18	19	20
1	50.59	51.96	53.20	54.33	55.36	56.32	57.22	58.04	58.83	59.56
2	14.39	14.75	15.08	15.38	15.65	15.91	16.14	16.37	16.57	16.77
3	9.72	9.95	10.15	10.35	10.52	10.69	10.84	10.98	11.11	11.24
4	8.03	8.21	8.37	8.52	8.66	8.79	8.91	9.03	9.13	9.23
5	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.21
6	6.65	6.79	6.92	7.03	7.14	7.24	7.34	7.43	7.51	7.59
7	6.30	6.43	6.55	6.66	6.76	6.85	6.94	7.02	7.10	7.17
8	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87
9	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.64
10	5.72	5.83	5.93	6.03	6.11	6.19	6.27	6.34	6.40	6.47
11	5.61	5.71	5.81	5.90	5.98	6.06	6.13	6.20	6.27	6.33
12	5.51	5.61	5.71	5.80	5.88	5.95	6.02	6.09	6.15	6.21
13	5.43	5.53	5.63	5.71	5.79	5.86	5.93	5.99	6.05	6.11
14	5.36	5.46	5.55	5.64	5.71	5.79	5.85	5.91	5.97	6.03
15	5.31	5.40	5.49	5.57	5.65	5.72	5.78	5.85	5.90	5.96

TABLE II (continued)

Error df	11	12	13	14	15	16	17	18	19	20
16	5.26	5.35	5.44	5.52	5.59	5.66	5.73	5.79	5.84	5.90
17	5.21	5.31	5.39	5.47	5.54	5.61	5.67	5.73	5.79	5.84
18	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79
19	5.14	5.23	5.31	5.39	5.46	5.53	5.59	5.65	5.70	5.75
20	5.11	5.20	5.28	5.36	5.43	5.49	5.55	5.61	5.66	5.71
24	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.49	5.55	5.59
30	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.47
40	4.82	4.90	4.98	5.04	5.11	5.16	5.22	5.27	5.31	5.36
60	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.15	5.20	5.24
120	4.64	4.71	4.78	4.84	4.90	4.95	5.00	5.04	5.09	5.13
∞	4.55	4.62	4.68	4.74	4.80	4.85	4.89	4.93	4.97	5.01

Upper 1% Points

Error df	2	3	4	5	6	7	8	9	10
1	90.03	135.0	164.3	185.6	202.2	215.8	227.2	237.0	245.6
2	14.04	19.02	22.29	24.72	26.63	28.20	29.53	30.68	31.69
3	8.26	10.62	12.17	13.33	14.24	15.00	15.64	16.20	16.69
4	6.51	8.12	9.17	9.96	10.58	11.10	11.55	11.93	12.27
5	5.70	6.98	7.80	8.42	8.91	9.32	9.67	9.97	10.24
6	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10
7	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37
8	4.75	5.64	6.20	6.62	6.96	7.24	7.47	7.68	7.86
9	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.33	7.49
10	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21
11	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99
12	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.67	6.81
13	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67
14	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54
15	4.17	4.84	5.25	5.56	5.80	5.99	6.16	6.31	6.44
16	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.22	6.35
17	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27
18	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20
19	4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14
20	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09
24	3.96	4.55	4.91	5.17	5.37	5.54	5.69	5.81	5.92
30	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76
40	3.82	4.37	4.70	4.93	5.11	5.26	5.39	5.50	5.60
60	3.76	4.28	4.59	4.82	4.99	5.13	5.25	5.36	5.45
120	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30
∞	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16